Fear and Citizen Coordination Against Dictatorship*

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September 4, 2018

Abstract
Despite numerous studies showing that emotions influence political decision making, there is scant literature giving a formal treatment to this phenomenon. This paper formalizes insights about how fear influences participation in risky collective action such as citizen revolt against an autocratic regime. To do so we build a global game and analyze the effects that fear may have on participation through increasing pessimism about the regime’s strength, increasing pessimism about the participation of others in the revolution, and increasing risk aversion. The impact of the first two effects of fear is a clear reduction in the probability that people will mobilize. However, an increase in risk aversion may in some circumstances increase the probability with which citizens will mobilize. These results may help explain the unpredictable reactions of citizens to fear appeals, including the threat of repressive violence.

*Thanks to Eric Dickson, Tiberiu Dragu, Andrew Little, Zhaotian Luo, Scott Tyson, the anonymous reviewers and seminar participants at the NYU Graduate Political Economy Lunch, SPSA, and MPSA for helpful comments and suggestions.
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Introduction

Emotions affect political behavior. A large literature in psychology has established that emotions such as fear, anger, and happiness affect beliefs, preferences, and decision-making, particularly in situations involving risk and cooperation (Damasio, 1994; LeDoux, 1996; Lerner and Keltner, 2000, 2001). These findings have subsequently sparked a large research agenda on American political behavior (Marcus et al., 2000; Brader, 2005; Valentino et al., 2011; Albertson and Gadarian, 2015), and have recently been applied to the study of participation in high-risk contentious politics (Young, 2015, 2016; Pearlman, 2016; Aytac et al., 2017).

However, despite the increasing influence that these findings in psychology have had on the empirical study of political science, the role of emotions has not been given much treatment in the formal theoretic literature (Elster, 1996), with a few important exceptions (Akerlof, 2016; Wu, 1999; Caplin and Leahy, 2001; Lupia and Menning, 2009). This omission is important for several reasons. First, given that emotions have been shown to have numerous effects on cognition, formal analysis may provide insight into the ways that emotions affect decisions in strategic situations. Second, a formal treatment may provide insights into the conditions in which emotional appeals might have particularly large effects, or counter-intuitive effects. Ultimately, this analysis may increase our understanding of how emotional appeals can be used by political elites to cause citizens to act against their long-run interests. This paper thus builds on efforts to incorporate behavioral assumptions into formal models such as Lupia and Menning (2009) and Little (2017) that have shown how deviations from the strict assumption of citizen rationality can have dramatic effects on the support for and longevity of a regime.

As part of a larger project in which we seek to understand the effect of fear on coordination, in this paper we build a model based on a global game (Carlsson and Van Damme, 1993; Morris and Shin, 2003, 2004) in which people decide whether or not to take a potentially costly action to overthrow a regime. Based on theory and empirical findings in psychology and behavioral economics, we analyze various mechanisms through which this emotion may influence a citizen’s...
decision to mobilize against the regime. In particular, we focus on two of the most empirically established effects of fear on decision-making: that it increases pessimism about risks and increases risk aversion. In our models, fear may increase pessimism about the strength of the regime; independently, it may increase pessimism about the number of other citizens who will participate in the mobilization; and finally, it may increase risk aversion. When people are aware that the other players are also in a state of fear, its demobilizing effects may compound as not only will players themselves be more pessimistic and risk averse, but they will expect the same effects to change the behavior of others.

However, our formal treatment also shows that fear does not necessarily reduce participation in risky actions, as previous research has suggested. Instead, although increased pessimism always reduces participation in dissent, the effects of fear through risk aversion are ambiguous and depend on the effect the citizens’ emotional state on their utility function. The overall effect of fear will be negative when the effects via pessimism (which always reduce mobilization) dominate, or when the negative utility of a failed mobilization is large relative to the status quo. However, when the effect on mobilization via risk aversion is relatively large, and when what we call a “nothing-to-lose effect” prevails, fear can actually make it more likely that citizens will mobilize. This result, while arguably a rare case, speaks to an important puzzle in the literature on repression and protest: why do autocrats use repression when in many cases it seems to backfire, setting off even larger cycles of protest (Davenport, 2007)? Our analysis suggests that fear may be a powerful but unpredictable tool for autocrats because its effects can depend on the balance of psychological processes that are hard for the autocrat to observe.

This project relates to a large literature on the importance of emotions for a range of political and economic behaviors, particularly participation in collective action, social sanctioning, and public goods provision. Past experimental work has shown that emotions may play an important role in social sanctioning (Reuben and Van Winden, 2008; Hopfensitz and Reuben, 2009), generosity (Kirchsteiger et al., 2006), and trust (Dunn and Schweitzer, 2005; Myers and Tingley, 2016). Much of this work has been focused on emotions thought to motivate prosocial contributions such as anger, guilt, and happiness. We seek to
understand whether the emotion of fear may decrease prosocial participation, in part because this may shed light on how emotions can be used strategically by powerful actors trying to demobilize citizens.

Second, this project relates to our understanding of citizen mass action. The role of emotions has been a key point of disagreement in the study of contentious politics (see, for example, Goodwin and Jasper (2004) for an overview of the debate). Formal treatments of collective action have largely relied on the implicit assumption that emotions do not systematically change the way that actors make decisions. We build on a literature that models citizens’ decisions to participate in attempts to install a democratic system of government as a function of the risks and benefits of mobilization, taking into account the behavior of other citizens. Contributors to this literature have used both a collective action framework in which citizens view their participation and others’ participation as substitutes (Cantoni et al., 2018), and a coordination framework, in which citizens’ participation decisions are complements, to analyze pro-democracy mobilization (Kuran, 1991). Recently, global games have been applied to the study of regime change to produce insights about how information quality or group size influence the size and frequency of pro-democracy protest (Angeletos et al., 2007; Edmond, 2013). We adopt the framework of a global game in which citizens’ participation has strategic complementarities, and incorporate some of the well-documented effects of fear on risk perceptions and attitudes into citizens’ decisions. Our approach illustrates a way to model strategic interactions between citizens and autocratic elites that builds in a more realistic set of assumptions about how citizens make decisions.

Finally, this project makes a contribution to a small collection of models in political science that attempt to formalize insights from psychology. Models addressing a variety of topics have simply added a term that captures the non-monetary or expressive benefits that individuals get from seemingly irrational behaviors like voting or low demand for social services (Riker and Ordeshook, 1968; Scheve and Stasavage, 2006). Others have relaxed the assumption that

\(^1\)Models that are similar to global games but without global strategic complementarities and two-sided dominance such as Bueno de Mesquita (2010) and Shadmehr and Bernhardt (2011) have also been used.
citizens are strategic for one period of a multi-period game (Lupia and Menning, 2009), or for some proportion of the population (Little, 2017), to model the effects of emotions or thoughtfulness of different voters. Other recent models have endogenized preferences to formalize the concept of cognitive dissonance (Acharya et al., 2018) or motivated reasoning (Little, 2018). We add to these efforts by showing how the effects of emotions on perceptions and basic preferences like risk aversion can be incorporated into a strategic game.

**Emotions and decisions about risk**

This section provides a brief overview of the research in psychology and neuroscience that motivates our models. Emotions are patterned chemical and neural responses that motivate behavior to deal with relevant events (Frijda, 1994; Damasio, 1994). Emotional responses are “brief, often quick, complex, organized, and difficult to control” (Ekman, 1977: 25). They occur in response to a stimulus, which in the case of primary emotions like fear activates the amygdala region of the brain. The amygdala then sets off a number of reactions in the brain and body in response to the input using neurotransmitters and chemical signals through the bloodstream like endocrine (Damasio, 1994).

Theory in both neuroscience and psychology highlights the way that emotions are distinct from, yet inextricably linked to, cognition. One theory of emotions in psychology known as cognitive appraisal theory posits that emotions are determined by cognitive appraisals of the state of the world in relation to one’s goals (Lazarus, 1991, 1994).² Smith and Ellsworth (1985) identify six dimensions of appraisals that underly emotional responses: certainty, pleasantness, attentional activity, control, anticipated effort, and responsibility. For example, fear is defined primarily by low certainty, low pleasantness, low control, and high anticipated effort (Lerner and Keltner, 2000).

Lerner and Keltner’s (2000; 2001) appraisal tendency theory (ATT) integrates insight from both cognitive appraisal theory focused on the cognitive antecedents of emotions and functional theories focused on how emotions enable evolutionarily

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²Nevertheless, Zajonc (1980) has shown using subconscious priming that these “cognitive” appraisals may occur pre-consciously.
advantageous responses to stimuli. Specifically, ATT posits that emotions are not only induced by cognitive appraisals, but also that “each emotion activates a cognitive predisposition to appraise future events in line with the central-appraisal dimensions that triggered the emotion” (Lerner and Keltner, 2000: 477). In other words, an individual’s emotional state should influence her perceptions of other information in a way that may reinforce an appropriate response. However, the effects of emotions are not specific to the particular situation that caused them.

There is strong empirical evidence in psychology and economics that emotions influence risk perceptions and risk aversion. In particular, there is experimental evidence that fear increases perceptions of risks (Johnson and Tversky, 1983; Lerner and Keltner, 2000, 2001; Lerner et al., 2003) and risk aversion (Guiso et al., 2013; Cohn et al., 2015). There is also some research in political science showing that fear affects perceptions of the risk of repression and risk aversion among opposition supporters living under autocracy (Young, 2016). We interpret this literature as suggesting various channels through which fear may be operating. First, if fear affects risk aversion, then it changes the concavity of the citizens’ utility functions.\(^3\) Second, if fear affects risk perceptions it should influence citizens’ beliefs of how strong the regime actually is or, independently from this, citizens’ beliefs about how likely it is that other citizens will participate in the revolution.

In sum, this project builds on the growing body of research in psychology, and more recently in the social sciences, about how emotions may affect decision-making about risk and cooperation. Pioneering political psychologist Rose McDermott argued that because political decisions are often “highly uncertain, ambiguous, and dynamic,” the political arena is exactly where we would expect biases in perceptions to play an important role (2001: 9). Our application of citizen participation in mass mobilization that might be met with repression is a prime example of that argument.

\(^3\)For a different but somewhat related discussion on the inclusion of risk aversion in global games see Goldstein and Pauzner (2004) and Guimaraes and Morris (2007).
Model

The Basic Model

We model the decision to join an anti-regime protest as a standard global game (Morris and Shin, 2004). This provides a framework similar to the one used in much of the recent formal literature on regime change (Angeletos et al., 2007; Bueno de Mesquita, 2010; Edmond, 2013; Shadmehr and Bernhardt, 2011), but that incorporates the role that fear may play in people’s decision.\footnote{Throughout the paper we assume that citizens cannot self-correct for the effect of fear; i.e. fear introduces biases. This could be produced by citizens not being aware of their own fear or the extent to which it affects their cognition, at least in the short term. Perhaps, in a multiple period game, people could learn to self-correct as is the case in Lupia and Menning (2009).} First we will begin with the basic components of the model that will be common to all of our treatments of fear. Consider a model in which a mass $z$ of citizens must decide whether to mobilize against an incumbent regime or to abstain from doing so. Denote the proportion of citizens that decide to mobilize as $l$. Depending on the (exogenously given) type of the regime, $\theta$, a mass $zl^*$ of the citizenry is required to mobilize in order to bring the regime down. We call the scenario in which the citizens overthrow the government a successful revolution as opposed to the status quo.

Formally, $\theta$ sets the threshold that the proportion of the citizens must exceed in order for a revolution to succeed. If $\theta \geq zl$ the regime survives. The regime’s type reflects characteristics as cohesiveness, competence, and strength. Citizens cannot observe the true values of $\theta$. However, they know that $\theta$ is randomly drawn from a normal distribution with mean $y$ and variance $1/\alpha$ (precision $\alpha$), i.e.,

$$\theta \sim N(y, 1/\alpha)$$

Citizens that decide to mobilize will receive particularlistic benefits in the case of a successful revolution. These can be understood as political favors or retributions from the new regime. This means that citizens are compensated for their participation in bringing down the ancien régime. When the revolution brings down the regime—a successful revolution (SR)—, meaning the government
Table 1: Payoff summary

<table>
<thead>
<tr>
<th>Citizens</th>
<th>SR</th>
<th>SQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstain</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Protest</td>
<td>$R$</td>
<td>$-T$</td>
</tr>
</tbody>
</table>

Type is sufficiently low that the revolt succeeds, citizens that mobilize receive a benefit normalized to $R$. However, mobilization is costly. If a citizen mobilizes and the regime is strong enough to stay in power, meaning that the status quo (SQ) prevails, the citizens that mobilize receive a payoff of $-T$, which captures any penalty that the regime may impose on dissidents. Finally, if a citizen decides to abstain, she gets a payoff of 0. Payoffs are summarized in Table 1.

Note, importantly, that all these payoffs refer to the material benefits that people receive from either abstaining or mobilizing. Once $\theta$ is realized, independent signals are privately drawn for each player. The signal each player receives is a noisy signal of $\theta$. In particular, each player receives a signal

$$x_i = \theta + \epsilon_i$$  \hspace{1cm} (1)

where $\epsilon_i$ is normally distributed with mean 0 and variance $1/\beta$.

Model 1: Increase in perceived strength of the regime

In this section we model citizens’ emotional state as affecting how they perceive the signal of the regime’s strength that they receive. This prediction operationalizes findings from psychology and political psychology showing that fear makes citizens more pessimistic, including about their personal risk of repression (Lerner and Keltner, 2000, 2001; Lerner et al., 2003; Young, 2016). It is important to note that results are substantively unchanged if we model pessimism as being a change in the mean of citizen’s prior belief about the regime’s strength.\footnote{Although we assume that the payoffs for abstaining are always 0, regardless of whether there is a successful revolution or not, this is only a simplifying assumption. Results are robust to having the payoff of a successful revolution and abstaining be $0 < B < R$, which we show in the appendix.}

\footnote{Modeling citizens as giving a higher weight than Bayes’ rule would warrant to a higher level of the signal or the prior would also render qualitatively the same results.}

Citizens that
are afraid behave pessimistically and act automatically as if they had received a signal $\hat{x}_i > x_i$. This means that the beliefs about the regime are affected by the citizens’ emotional state. In other words, each citizen $i$ acts as if she received a private signal

$$\hat{x}_i = \theta + F + \epsilon_i$$

where $F$ is a parameter that measures the amount of fear the citizen experiences.\(^7\)

Given that citizens do not observe $\theta$ directly, once they observe their signal, they base their decision to participate or abstain on their expectations about $\theta$ (and hence about the likely actions of other citizens), which depend on their emotional state. Fearful citizens act as if they received stronger signals. Citizens’ posterior beliefs about the regime type are derived by Bayes’ rule. In particular, given a perceived signal $\hat{x}_i$, a citizen’s posterior expectation of $\theta$ is normally distributed with mean $\gamma$ and precision $\alpha + \beta$, where $\gamma = \frac{\alpha y + \beta \hat{x}_i}{\alpha + \beta}$.

In this case, the equilibrium consists of the cutoff for dissidents $x^*$, and a critical threshold for revolution success $\theta^*$.\(^8\) Citizen’s using a cutoff strategy will choose to mobilize if they believe the regime is sufficiently weak and will abstain if they believe it is sufficiently strong. In particular there will be a switching point $\gamma^*$ at which a citizen will be indifferent between mobilizing, which is given by the true signal

$$x^* = \frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta} - F.$$  

(2)

Using cutoff strategies, citizens who receive a private signal $x$ will mobilize if $x \leq x^*$, and abstain otherwise. For a citizen cutoff $x^*$, the proportion of dissidents who mobilize, as a function of $\theta$ is $l = Pr(x \leq x^*|\theta) = \Phi(\sqrt{\beta}(x^* - \theta))$, where $\Phi$ is the cdf of a standard normal distribution. Recall that a regime survives if $\theta \geq zl$ and falls otherwise. Consequently, for the signal $x^*$, the critical type of a regime, the regime just on the cutoff between surviving and falling, is given by:

$$\theta^* = z\Phi\left(\sqrt{\beta}\left(\frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta} - F - \theta^*\right)\right).$$  

(3)

\(^7\)Equivalently we could say that under fear citizens’ signals are derived from a a distribution with mean $\theta + F$ but that they believe that they are derived from a distribution with mean $\theta$.

\(^8\)For a more lengthy discussion of the equilibrium of this game see Morris and Shin (2004).
When should an individual join the mobilization against the regime? To answer this question, consider first an alternative simpler setting in which the type of the government is public information and thus common knowledge among all citizens. If $\theta < 0$, each citizen prefers to mobilize, regardless of how many people plan to mobilize. Alternatively, if $\theta > z$ abstaining strictly dominates mobilizing. When the value is in the interval $(0,z)$, however, mobilizing is a coordination problem. Whether the group succeeds depends crucially on the beliefs about the intended actions of the others. In the case at hand, with incomplete information, citizens will mobilize or not depending on the signals they receive and the payoffs in case of a successful revolution and the costs of participating in a failed revolution.

In particular, the expected payoff for mobilizing for the citizen who receives signal $x_i$ is

$$R(Pr(\theta \leq \theta^*|x_i)) - T(1 - Pr(\theta \leq \theta^*|x_i))$$

and the payoff to abstain is 0. In particular it must be the case for a citizen who observes the signal $x_i = x^*$ that she is “indifferent” between mobilizing and abstaining.\(^9\) Hence we have

$$Pr(\theta \leq \theta^*|x^*) = \frac{T}{R+T}$$

which turns into

$$\Phi(\sqrt{\alpha + \beta(\theta^* - \gamma^*)}) = \frac{T}{R+T}.$$ 

Rearranging, this yields the citizens’ “indifference” posterior:

$$\gamma^* = \theta^* - \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1} \left( \frac{T}{R+T} \right). \tag{4}$$

We can now substitute this “indifference” posterior into Equation 3:

$$\theta^* = z\Phi \left( \frac{\alpha}{\sqrt{\beta}} \theta^* - \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \Phi^{-1} \left( \frac{T}{R+T} \right) - \frac{\alpha}{\sqrt{\beta}} y - \sqrt{\beta} F \right) \tag{5}$$

\(^9\)Rather than being indifferent, it must be the case that she believes that she is indifferent.
Note that equation (5) has a unique solution if the derivative of the right hand side with respect to $\theta^*$ is less than one in every possible case. This derivative is given by $z\phi(\cdot)$, where $\phi(\cdot)$ is the pdf of a standard normal, which is bounded above by $1/\sqrt{2\pi}$. Hence if $\alpha/\sqrt{\beta} < \sqrt{2\pi}/z$ (Assumption 1), which we will assume for the rest of the paper, a unique solution is guaranteed.

We are now in a position to do comparative statics on the amount of fear that a citizen experiences. In particular we have the following results:

**Proposition 1.** Increasing fear reduces the critical value of the regime that survives, $\theta^*$.

**Proof.** In Equation (5), take the derivative of $\theta^*$ with respect to $F$ on both sides of the equation. This leaves:

$$\frac{\partial \theta^*}{\partial F} = z\delta \phi(\cdot) \left( \frac{\alpha}{\sqrt{\beta}} \frac{\partial \theta^*}{\partial F} - \sqrt{\beta} \right)$$

Solving for $\frac{\partial \theta^*}{\partial F}$, we obtain

$$\frac{\partial \theta^*}{\partial F} = -\frac{\sqrt{\beta}(z\phi(\cdot))}{1 - z\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}$$

By assumption 1, $\phi(\cdot)\frac{\alpha}{\sqrt{\beta}} < 1/z$, and hence $\frac{\partial \theta^*}{\partial F} < 0$. ■

It is straightforward to note from equation 3 that since $\frac{\partial \theta^*}{\partial F} < 0$, then it follows that $\frac{\partial x^*}{\partial F} < 0$. This leads us to our second result:

**Proposition 2.** Increasing fear reduces the critical value of the signal, $x^*$, at which citizens stop mobilizing.

**Proof.** Follows directly from substituting equation (4) into equation (2) and taking the derivative with respect to $F$. The rest of the proof is given by the argument in Proposition 1. ■
Model 2: Increased pessimism about the participation of other citizens

The discussion so far underscores the importance of the effect of fear on two types of uncertainty: *fundamental uncertainty* and *strategic uncertainty*. Fundamental uncertainty refers to uncertainty concerning the payoff relevant state of nature, denoted by the regime’s strength, $\theta$. Strategic uncertainty refers to the uncertainty concerning the actions of others. This is an important distinction: Myatt et al. (2002) show that in some cases even as fundamental uncertainty becomes smaller, strategic uncertainty can remain large, and therefore the two types of uncertainty should be considered separately.

In the previous model we considered the relationship between fear and collective action exploring how fear increases the perceived strength of the regime. In other words, Model 1 analyzed the effects of fear on fundamental uncertainty. In this section we isolate the effect of fear on strategic uncertainty. We study to what extent the effect of fear on mobilization and regime survival is driven by changes in expectations about the actions of others.

In this model, we assume that each player believes that everyone will only mobilize when optimal to do so with some probability that is decreasing with the level of fear they experience. As we show below, it does not matter for citizens’ decisions whether they think that they themselves are prone to making mistakes or whether they only think that others will fail to behave optimally. While this is a deviation from the standard assumptions of formal models of coordination, both cases have base in the psychology literature. On the one hand we can interpret fear as having certain actions tendencies, in this case that of avoiding danger (Lazarus, 1991). As fear increases people are more likely to want to avoid danger and believe others will too even if they should optimally not. On the other hand, the case in which citizens believe others will be prone to not taking an optimal action has a basis in the psychology literature on rosy self-perceptions and over-confidence.\(^\text{10}\) One interpretation of the way that we model the strategic uncertainty of fear is that it enhances the common tendency to see others as less rational, brave, or strong-willed than we see ourselves. This argument requires

\(^\text{10}\)See, for example, Taylor and Brown (1988, 1994) on overly positive self-evaluations.
that each player places sufficient probability on the event that others will fail to behave optimally. In particular, past empirical research suggests that fear may make citizens more pessimistic about how others will behave (Young, 2016).

In this model, each citizen receives a signal $x_i = \theta + \epsilon_i$ of the strength of the regime. From the previous section we know that citizens update their belief about $\theta$ using Bayes’ rule, and that the posterior distribution is given by by $\theta|x_i \sim N(\gamma, \frac{1}{\alpha+\beta})$, where $\gamma = \frac{\alpha y + \beta x_i}{\alpha+\beta}$.

If citizens use a cutoff strategy there is a switching point $\gamma^*$ for the updated mean of the distribution at which citizens will be indifferent between mobilizing and not. Then, there will also exist a signal $x^*$ at which citizens are indifferent between both actions:

$$x^* = \frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta}.$$  (6)

The perceived probability of a successful mobilization depends on the beliefs about the actions of others given a signal $x_i$. In a context in which fear increases pessimism about other citizens’ (and potentially people’s own) actions, the presumed size of the revolution will be lower if citizens feel afraid. We model this as people believing that there is a tremble in the decision to mobilize; when being afraid people believe that others might make mistakes. As discussed below whether this belief is true or not render qualitatively similar results. If the beliefs are incorrect, however, we must also assume that citizens behave as if they were correct.

Formally, citizens believe that, upon receiving a signal that would make them rationally mobilize, everyone is only making a correct decision to mobilize with probability $\delta$. In this context, fear reduces $\delta$. This means that citizens believe that in equilibrium only a proportion $\delta$ of citizens that receive a signal $x < x^*$ do in fact mobilize.\(^{12}\) Hence, citizens posit that the proportion of dissidents who

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\(^{11}\)This prediction is also related to Little’s (2017) model showing that if citizens expect that others will believe the propaganda of an authoritarian regime, incentives to coordinate with other citizens may lead them to behave as if they believe the propaganda in support of the government.

\(^{12}\)It is possible that $\delta$ may be an increasing function of $x_i$, the signal that each citizen receives. This would imply that people that upon being afraid, citizens that believe that the regime is stronger are more likely to believe that others will “make mistakes”. This assumption would
mobilize, as a function of $\theta$ is now $l = \delta Pr(x \leq x^*|\theta) = \delta \Phi(\sqrt{\beta}(x^* - \theta))$. The regime that citizens believe is just on the cutoff between surviving and falling is given by

$$\theta^* = z\delta \Phi\left(\sqrt{\beta}\left(\frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta} - \theta^*\right)\right) \tag{7}$$

The expected payoff for choosing to mobilize (note that citizens believe that they will only implement their choice to mobilize with probability $\delta$) for a citizen that receives signal $x_i$ is now given by:

$$\delta(R(Pr(\theta \leq \theta^*|x_i)) - T(1 - Pr(\theta \leq \theta^*|x_i))) + (1 - \delta)0$$

The indifferent citizen is given by:

$$\Phi(\sqrt{\alpha + \beta}(\theta^* - \gamma^*)) = \frac{T}{R + T}$$

Note that $\delta$ drops out when calculating the indifferent citizen. This means that when calculating whether or not to mobilize, the indifferent citizen is not affected by the impact of $\delta$ on her own perceived probability of making a mistake. Hence, in equilibrium, citizens only take into account how much they expect others to be error prone because of fear and not the effect of fear on their own probability of making mistakes when choosing to mobilize. Thus, in this context, it does not matter whether citizens are actually prone to making mistakes when deciding to mobilize. Only the belief that other citizens are prone to making mistakes influences citizens’ decisions to mobilize.

The mean of the posterior at which citizens are “indifferent” is now given by:

$$\gamma^* = \theta^* - \frac{1}{\sqrt{\alpha + \beta}}\Phi^{-1}\left(\frac{T}{R + T}\right) \tag{8}$$

We can now calculate the perceived marginal regime, $\theta^*$, by substituting equation (8) into equation (7) render qualitatively the same results as our model, only making the effect stronger for people that were more likely not to participate anyway and weaker for those who were more likely to participate.
\[
\theta^* = z\delta\Phi\left(\frac{\alpha}{\sqrt{\beta}}\theta^* - \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}}\Phi^{-1}\left(\frac{T}{R+T}\right) - \frac{\alpha y}{\beta}\right) 
\]

(9)

**Lemma 1.** A decrease in the perceived probability of mobilizing given a signal below the threshold, \(\delta\), increases the value of the perceived critical regime.

*Proof.* Take the derivative of \(\theta^*\) in Equation (9) with respect to \(\delta\):

\[
\frac{\partial \theta^*}{\partial \delta} = z\Phi(\cdot) + z\delta\phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial \delta}\right)
\]

Once we solve for \(\frac{\partial \theta^*}{\partial \delta}\), we obtain

\[
\frac{\partial \theta^*}{\partial \delta} = \frac{z\Phi(\cdot)}{1 - z\delta\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}
\]

Recall that we assumed that \(\phi(\cdot)\frac{\alpha + \beta}{\sqrt{\beta}} < 1/z\) and note that the CDF of a Normal distribution is positive. Therefore \(\frac{\partial \theta^*}{\partial \delta} > 0\). ■

Since \(\frac{\partial \theta^*}{\partial \delta} > 0\), it follows that \(\frac{\partial \gamma^*}{\partial \delta} > 0\), from which the next result obtains:

**Proposition 3.** A reduction in the perceived proportion of people mobilizing, increases the critical value of the signal \(x^*\) at which citizens stop mobilizing.

*Proof.* The results follows from substituting equation (8) into equation (6), taking the derivative with respect to \(\delta\) and the argument in Lemma 1. ■

If citizens’ beliefs are in fact true, and fear indeed makes people more error prone, we are done. In this case, the perceived critical regime in fact corresponds to the true critical regime. If however, citizens’ beliefs are incorrect, we still have to obtain the true critical regime, \(\theta_{true}^*\). It is worth noting that in calculating \(\theta_{true}^*\), we do not multiply the proportion of people who decide to mobilize by \(\delta\). The following result shows that it too decreases by increasing fear.

**Proposition 4.** An increase in \(\delta\) leads to an increase in the true value of the...
critical regime.

**Proof.** The true critical regime is given by

\[
\theta_{\text{true}}^* = z \Pr(x \leq x^* | \theta)
\]

which substituting in the value of \( \Pr(x \leq x^* | \theta) \) found above becomes

\[
\theta_{\text{true}}^* = z \Phi \left( \frac{\alpha \theta^* - \sqrt{\alpha + \beta} \Phi^{-1} \left( \frac{T}{R + T} \right) - \frac{\alpha y}{\beta}}{\sqrt{\beta}} \right)
\]

Taking the derivative with respect to \( \delta \) we obtain

\[
\frac{\partial \theta_{\text{true}}^*}{\partial \delta} = z \phi \left( \cdot \right) \frac{\alpha}{\sqrt{\beta}} \frac{\partial \theta^*}{\partial \delta}
\]

which by Lemma 1 is clearly positive. ■

In the case in which fear reduces the perceived probability of a successful mobilization, citizens are less willing to mobilize. In this sense, pessimism is then a self fulfilling prophecy. If everyone believes that everyone will be less willing to mobilize against the regime, then everyone will be less willing to mobilize against the regime.

**Model 3: Increase in risk aversion**

So far in all of our modeling choices we have not made any explicit assumption about citizens’ risk preferences. However, as noted above, there is experimental evidence in economics and in cognitive and political psychology that fear may increase risk aversion (Young, 2016; Guiso et al., 2013; Cohn et al., 2015). In this section we model fear as having an effect on the concavity of the citizens utility function. In order to do this, suppose now that citizens have utilities over the material payoffs they receive from this game \( u(\mu, P) \), where \( P \) is the material payoff and \( \mu \) is a parameter that measures the amount of fear that citizens experience. Moreover for mnemonic purposes suppose that in the case without fear a material payoff of \(-t\) corresponded to a utility of \(-T\), i.e. \( u(0, -t) = -T \),
and that $u(0, o) = 0$ and $u(0, r) = R$, where $-t < o < r$.\footnote{These terms then simplify to the payoffs specified in Table 1.} We assume that the citizens’ utility function is twice continuously differentiable in $P \in (-T, \infty)$ and at least once continuously differentiable in $\mu$ and that it is increasing and concave in the material payoffs, thus $\frac{\partial u}{\partial P} > 0$ and $\frac{\partial^2 u}{\partial P^2} < 0$. For simplicity and without loss of generality we also assume that $u(\mu, -t) = -T$ for any level of $\mu$, however for all other levels of $P$ we assume that $\frac{\partial u}{\partial \mu} < 0$.\footnote{For our purposes this means that $u(\mu, o) < 0$ for $\mu > 0$ and that $u(\mu, r) < R$ for $\mu > 0$.} This assumption is not crucial and only simplifies calculations. These payoffs are summarized in Table 2.

Consistent with the idea that fear makes citizens more risk averse and thus their utility more concave, we suppose that fear increases the Arrow-Pratt absolute risk aversion coefficient, which we will call $A$, and which we define as

$$A \equiv -\frac{\partial^2 u}{\partial P \partial \mu}.$$

Hence

$$\frac{\partial A}{\partial \mu} = \left( -\frac{\partial^3 u}{\partial P^2 \partial \mu} \frac{\partial u}{\partial P} \right) - \left( -\frac{\partial^2 u}{\partial P^2} \frac{\partial^2 u}{\partial \mu \partial P} \right) \left( \frac{\partial u}{\partial P} \right)^2 > 0$$

In order for this equation to hold it must be that the numerator is positive. Under our assumptions it must be then that at least one of the following two hold: (1) $\frac{\partial^3 u}{\partial P^2 \partial \mu} < 0$ or (2) $\frac{\partial^2 u}{\partial \mu \partial P} < 0$ for all $P$. Condition (1) has a complicated interpretation, this term measures the change that fear has on the second derivative of the citizens’ utility function, if the condition is met it means that at higher values of $\mu$ the second derivative is more negative. Condition (2), which is illustrated in Figure 1, has a more straightforward interpretation. If it holds, then at larger values of material payoffs the effect that fear has on utility is larger. In the analysis that follows, we do not need to use Condition (1) again, so to make the

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
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<td>$u(\mu, o)$</td>
<td>$u(\mu, o)$</td>
</tr>
<tr>
<td>Protest</td>
<td>$u(\mu, r)$</td>
<td>$-T$</td>
</tr>
</tbody>
</table>

Table 2: Payoffs with risk aversion
In this graph $u(P, \mu) = (1 + T + P)(\frac{3}{4} - \mu)$. In the condition without fear $\mu = 0$; in the fear condition $\mu = \frac{1}{4}$.

Figure 1: Larger Effect of Fear at Higher Levels of Material Payoffs

analysis more interesting we will assume that Condition (2) holds (Assumption 2).

Given Assumption 2, we can now solve the model. Recall that citizens receive a signal $x$ of the strength of the regime, $\theta$. Given this signal and using Bayes’ rule, they update their beliefs about $\theta$. Their posterior distribution is given by $\theta|x_i \sim N(\gamma, \frac{1}{\alpha+\beta})$, where $\gamma = \frac{\alpha y + \beta x_i}{\alpha+\beta}$.\(^{15}\)

Similar to what we observe in the other two models, if citizens use a cutoff strategy, there is a switching point $\gamma^*$ for the updated mean of the distribution at which citizens will be indifferent between mobilizing and not. Then, there will also be a signal $x^*$ at which citizens are indifferent between both actions.

\(^{15}\)Note that although we use similar notation to that in model 1, the value of $\gamma$ is different.
\[ x^* = \frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta}. \]  

(10)

Recall that for a citizen cutoff \( x^* \) the proportion of dissidents who mobilize, as a function of \( \theta \) is \( l = Pr(x \leq x^*|\theta) = \Phi(\sqrt{\beta}(x^* - \theta)) \), where \( \Phi \) is the cdf of a standard normal distribution. Recall that a regime survives if \( \theta \geq zl \) and falls otherwise. Consequently, for the signal \( x^* \), the critical type of a regime, the regime just on the cutoff between surviving and falling, is given by:

\[ \theta^* = z\Phi\left(\sqrt{\beta}\left(\frac{(\alpha + \beta)\gamma^*}{\beta} - \frac{\alpha y}{\beta} - \theta^*\right)\right) \]  

(11)

We now evaluate the decision for the citizen that is indifferent between mobilizing and abstaining. Citizens that are indifferent will receive the same expected utility from mobilizing and abstaining. Given the discussion provided at the beginning of this section, formally this means that

\[ u(\mu, r)(Pr(\theta \leq \theta^*|x^*)) - T(1 - Pr(\theta \leq \theta^*|x^*)) = u(\mu, o) \]

Note as above that \( Pr(\theta \leq \theta^*|x^*) = \Phi(\sqrt{\alpha + \beta}(\theta^* - \gamma^*)) \). Hence rearranging terms we have that

\[ \Phi(\sqrt{\alpha + \beta}(\theta^* - \gamma^*)) = \pi. \]

where

\[ \pi \equiv \frac{u(\mu, o) + T}{u(\mu, r) + T}. \]

The posterior expected value of \( \theta \) at which citizens are indifferent is then given by

\[ \gamma^* = \theta^* - \frac{1}{\sqrt{\alpha + \beta}}\Phi^{-1}(\pi). \]  

(12)

We can now substitute this indifference posterior into Equation (11) in order to obtain the marginal regime:
We can now calculate the effect of fear on regime failure and mobilization. As proposition 5 shows, the effect is ambiguous.

**Proposition 5.** If fear increases risk aversion, the effect of fear on the strength of the critical regime will be ambiguous.

**Proof.** Take the derivative of $\theta^*$ with respect to $\mu$ on both sides of Equation (13).

\[
\frac{\partial \theta^*}{\partial \mu} = z\Phi(\cdot) \left( \frac{\alpha}{\sqrt{\beta}} \frac{\partial \theta^*}{\partial \mu} - \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \frac{\partial \Phi^{-1}}{\partial \pi} \frac{\partial \pi}{\partial \mu} \right)
\]

Solving for $\frac{\partial \theta^*}{\partial \mu}$, the following is obtained

\[
\frac{\partial \theta^*}{\partial \mu} = \frac{-\sqrt{\alpha + \beta} \Phi^{-1} \frac{\partial \pi}{\partial \mu} z\Phi(\cdot)}{1 - z\Phi(\cdot) \frac{\alpha}{\sqrt{\beta}}}
\]

Note as above, that since the inverse of a Normal cdf is increasing, the sign that the expression takes is the the opposite sign that $\frac{\partial \pi}{\partial \mu}$. Taking the derivative of $\pi$ from its definition, we have the following result:

\[
sign \left( \frac{\partial \theta^*}{\partial \mu} \right) = -\text{sign} \left( \frac{\partial u(\mu,o)}{\partial \mu} (u(\mu,r) + T) - \frac{\partial u(\mu,r)}{\partial \mu} (u(\mu,o) + T) \right) \frac{1}{(u(\mu,r) + T)^2} \]

(14)

Rearranging terms we have

\[
sign \left( \frac{\partial \theta^*}{\partial \mu} \right) = -\text{sign} \left( \frac{\partial u(\mu,o)}{\partial \mu} - \frac{\partial u(\mu,r)}{\partial \mu} \frac{u(\mu,r) - u(\mu,o)}{(u(\mu,r) + T)^2} \right) \]

(15)

Note that since we have assumed that $\frac{\partial u}{\partial \mu} < 0$, $\frac{\partial^2 u}{\partial \mu \partial P} < 0$ for all $P$, then $|\frac{\partial u(\mu,o)}{\partial \mu}| < |\frac{\partial u(\mu,r)}{\partial \mu}|$. Note also that since citizens' utility is increasing in the ma-
terial payoff that \( u(\mu, r) > u(\mu, o) > -T \). Finally, note that \( u(\mu, o) < 0 \) and that \( u(\mu, r) < R \); note however that this does not imply what the sign of \( u(\mu, r) \) is.\(^{16}\) Hence, given this discussion, if we analyze the terms of the numerator in the RHS of Equation (15) we find that the first term does not have a clear sign, the second term is clearly negative and the third term is clearly positive. Then the effect that an increase on fear has on \( \theta^* \) is ambiguous. ■

Similarly to the models in the two previous sections we can now show the effect of fear on the critical signal. Not surprisingly it is also ambiguous.

**Proposition 6.** The effect of fear, if it increases risk aversion, is ambiguous on the critical signal \( x^* \).

**Proof.** Follows directly from the fact that from Proposition 5 we know that the effect of fear over \( \theta^* \) is ambiguous and from substituting the value of \( \gamma^* \) from equation 12 into equation 10 and deriving it with respect to \( \mu^* \). ■

The finding that fear has an ambiguous effect on dissent through risk aversion is counter-intuitive. It stems from the fact that an increase in risk aversion may cause \( \pi \), which is defined as the ratio of \( u(\mu, o) \) —the payoff to abstain— plus \( T \) —the negative of the payoff to protest and having the status quo remain in place— and \( u(\mu, r) \) —the payoff to protest and having a successful revolution— plus \( T \), to increase. This can be observed in the RHS of equation (14), which shows the sign of the derivative of \( \pi \) with respect to fear. When this term is positive, the effect of fear through risk aversion will have a mobilizing effect. Given that the denominator is a squared term, it is clearly positive. Moreover, since the derivative of the utility function with respect to fear is always negative and it must be the case that \( u(\mu, r) > -T \), the first term of the numerator must be negative. If \( u(\mu, o) \approx -T \), then the second term will be approximately zero, and then the effect of fear on the critical regime through risk aversion will be positive; hence, under these conditions fear will increase the strength threshold.

---

\(^{16}\)This assumption is not crucial, in order for the sign of subtraction of the first two terms to be indeterminate we just need \( \frac{\partial^2 u}{\partial \mu \partial \psi} < 0 \) and \( u(\mu, r) > u(\mu, o) \) to hold.
below which a regime will collapse, essentially making citizens more likely to mobilize. The overall effect of fear will be positive when this effect through risk aversion is positive, and dominates the effects on pessimism outlined in models 1 and 2.

Substantively, this model suggests that one of the effects of fear is to make citizens discount the status quo, or believe that they have nothing to lose by mobilizing. Although citizens get a higher payoff in the status quo than if they are repressed, an increase in risk aversion causes them to get relatively less utility from the status quo compared to an unsuccessful revolution in which they face repression.\textsuperscript{17}

**Discussion**

Machiavelli famously advised Lorenzo de Medici that it is “much safer to be feared than loved, when, of the two, either must be dispensed with” (1532, XVII). This has often been interpreted as an unambiguous endorsement of repression. He also, however, recognized that ruling by fear involves a trade off between using enough “inhuman cruelty” to be “revered and terrible”, without creating a backlash from citizens. Authoritarian regimes, which maintain control by excluding a sizable proportion of the citizenry from power, continue to struggle with the risk that repression might end up mobilizing their citizens against them, particularly when emotions make citizen decision-making less predictable. Machiavelli’s early analysis seems to recognize that citizens’ decisions to revolt are inherently affective. Yet, modern research on protest and authoritarian persistence has tended to analyze the decisions of citizens facing the threat of repression without considering how the actual emotion of fear might affect their willingness to take action against the repressive regime.

Our model takes an important step towards bringing the emotion of fear into the study of authoritarian politics. Research in psychology and behavioral economics has consistently found that the emotion of fear affects how individ-

\textsuperscript{17}One possible extension of our model would introduce the idea of prospect theory and loss aversion. To the extent that citizens view repression as a loss and have utility functions that are convex in the domain of losses, the “nothing-to-lose” effect would not hold.
uals perceive and process information, particularly information about risks in social environments. We incorporate the effects of fear on judgment and decision-making into a model of mass action against a repressive regime in three ways. First, we model how fear might increase citizen pessimism about the regime’s strength. Second, we model the effect of fear on pessimism about whether others will participate in dissent. Finally, we model the effect of fear on risk aversion.

Formalizing the effects of fear in this way may shed light on why repression is such an unpredictable tool for autocratic regimes. While the first two effects of fear through pessimism unambiguously reduce participation in dissent, the effect of fear through risk aversion may, under certain circumstances, actually increase mobilization. In cases where citizens get similar utility from the status quo and an unsuccessful revolution, an increase in risk aversion could push them into mobilization by discounting the utility that they get from the status quo. As a result, determining the overall effect of fear on mobilization requires assessing the magnitude of each of the three effects, and the direction of the effect of risk aversion. While under most circumstances it seems reasonable to think that the effects of fear on pessimism will dominate, when the effect of risk aversion is positive and large fear may actually be mobilizing.

This combination of factors that results in fear increasing mobilization may be particularly likely when citizens are almost indifferent between the status quo and being repressed. This is likely when citizens get little utility from the status quo and when the threat of repression associated with an unsuccessful mobilization is light. Considering that repression is often used by under-performing, unpopular regimes, it is easy to believe that many citizens facing the choice to mobilize against a frightening, repressive regime will perceive the utility associated with the status quo as quite low. Indeed, the “nothing-to-lose” effect identified in our model has echoes in numerous case studies of pro-democracy protests. Jim Scott’s (1977) “moral economy of the peasant”, for instance, suggests that once peasants reach a certain level of deprivation, they are no longer sensitive to the risk of repression. Other accounts of mobilization against the regime in Chile (Salman, 1994), Zimbabwe (Young, 2015), and the Soviet Union (Lohmann, 1993) emphasize the importance of citizens who discount the status quo after fear appeals like repressive violence. Our model suggests that this “nothing to lose”
mentality may be enhanced by the effect of fear on risk aversion.

This project also opens at various possible avenues for further research. On the one hand, one can study the relationship between other type of emotions and political participation. An obvious next step is to consider the inclusion of other emotions such as anger and empathy. On the other hand, in all the models we assumed that fear was exogenously introduced. A future approach may include a regime that can endogenously decide whether to induce fear in the population. In light of our results, the regime’s decision is particularly interesting because inducing fear could backfire and empower citizens to mobilize against the regime. Finally, another promising way forward could be to treat the status quo as a reference point for citizens, and consider that they might have different attitudes towards risk when they are in the domain of losses (such as the risk of repression relative to the status quo) instead of the domain of gains (such as the potential benefits of a successful revolution).
A Appendix

In this section we show that the results of the model hold when we change the payoff of abstaining from 0 to \(0 < B < R\), where \(B\) represents the payoff for abstaining in case of a successful revolution. Payoffs are now summarized as follows:

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>Abstain</td>
<td>(B)</td>
<td>0</td>
</tr>
<tr>
<td>Protest</td>
<td>(R)</td>
<td>(-T)</td>
</tr>
</tbody>
</table>

Table 3: Payoff summary with abstaining payoff \(B\)

We will consider the variation in each of the three models. In model 1, the new expected payoff for mobilizing for the citizen who receives the signal \(x\) is

\[
R(Pr(\theta \leq \theta^* | x)) - T(1 - Pr(\theta \leq \theta^* | x)),
\]

while the payoff to abstain is

\[
B(Pr(\theta \leq \theta^* | x)).
\]

We then have that for the indifferent citizen

\[
Pr(\theta \leq \theta^* | x^*) = \frac{T}{R - B + T},
\]

and conclude that the new critical type of a regime, \(\theta^*\) is given by

\[
\theta^* = z\Phi\left(\frac{\alpha \theta^* - \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}}\Phi^{-1}\left(\frac{T}{R - B + T}\right) - \frac{\alpha}{\sqrt{\beta}y - \sqrt{\beta}F}}{\sqrt{\beta}}\right) - \frac{\alpha}{\sqrt{\beta}y - \sqrt{\beta}F}\right) \quad (16)
\]

As a consequence, the effect of fear \(F\) on the regime, \(\theta^*\) and the critical value of the signal, \(x^*\) is analogous to the one proved in Proposition 1 and Proposition 2 respectively. Increasing fear reduces the critical value of the regime that survives, and reduces the critical value of the signal. The proofs follow directly from the argument in Proposition 1 and Proposition 2.

For model 2, the expected payoff of mobilizing is now given by
\[
\delta(R(Pr(\theta \leq \theta^*|x)) - T(1 - Pr(\theta \leq \theta^*|x))) + (1 - \delta)(BPr(\theta \leq \theta^*|x))
\]

and the payoff for abstaining is

\[
BPr(\theta \leq \theta^*|x).
\]

The indifferent citizen is now:

\[
\Phi(\sqrt{\alpha + \beta(\theta^* - \gamma^*)}) = \frac{T}{R - B + T}
\]

The new critical regime is given by

\[
\begin{align*}
\theta^* &= z\delta \left( \frac{\alpha}{\sqrt{\beta}} \theta^* - \sqrt{\frac{\alpha + \beta}{\beta}} \Phi^{-1}\left( \frac{T}{R - B + T} \right) - \frac{\alpha y}{\beta} \right)
\end{align*}
\] (17)

As in Model 1, changing the payoff for abstaining does not change the direction of the effect of fear on the critical regime nor on the critical value of the signal. The statements proved in Lemma 1, Proposition 3, and Proposition 4 remain unchanged.

Finally, in Model 3 we consider the effect of fear through risk aversion. The payoffs with risk aversion are represented as follows:

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</tr>
<tr>
<td>Protest</td>
<td>(u(\mu, r))</td>
<td>(-T)</td>
</tr>
</tbody>
</table>

Table 4: Payoffs with \(B\) and with risk aversion

As in the main corpus of the paper, suppose that in the case without fear a material payoff of \(b\) corresponded to a utility of \(B\), \(u(0, b) = B\). We keep the same assumptions on the utility function.

Let

\[
\pi' = \frac{u(\mu, o) + T}{u(\mu, r) - u(\mu, b) + T + u(\mu, o)}
\]

Then the new critical value of \(\theta\) is given by
\[ \theta^* = z \Phi \left( \frac{\alpha}{\sqrt{\beta}} \theta^* - \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \Phi^{-1}(\pi') - \frac{\alpha}{\sqrt{\beta}} \theta \right) \]  

(18)

which is a clear analogue of equation (13) in the main paper. Similarly to our result in Proposition 5, the sign of \( \frac{\partial \theta^*}{\partial \mu} \) will be the same as that of \( \frac{\partial \pi'}{\partial \mu} \). Note that

\[
\frac{\partial \pi'}{\partial \mu} = \frac{\partial u(\mu,o)}{\partial \mu} (u(\mu,r) + T - u(\mu,b)) - \left( \frac{\partial u(\mu,r)}{\partial \mu} - \frac{\partial u(\mu,b)}{\partial \mu} + \frac{\partial u(\mu,o)}{\partial \mu} \right) (u(\mu,o) + T)
\]

\[
\frac{\partial u(\cdot)}{\partial \mu} (u(\mu,r) - u(\mu,b) + T + u(\mu,o))^2
\]

Since the denominator is a squared number, it is positive. We know that \( \frac{\partial u(\cdot)}{\partial \mu} < 0 \) and that \(-T < u(\mu,b) < u(\mu,r)\) and thus the first term of the numerator is negative. The second term has two components, the first of which \( \frac{\partial u(\mu,r)}{\partial \mu} - \frac{\partial u(\mu,b)}{\partial \mu} + \frac{\partial u(\mu,o)}{\partial \mu} \) is negative, because \( \frac{\partial u(\mu,r)}{\partial \mu} < \frac{\partial u(\mu,b)}{\partial \mu} \). The second component of the second term is clearly positive, making the second term negative. With the subtraction it becomes positive and added to the first term there is no way to assess which one is larger, making the direction of the derivative indeterminate.
References


