A model of cultural transmission, voting and political ideology

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Abstract

In this paper, we present a model of cultural transmission of preferences on goods, some of which are provided publicly through simple majority voting. We emphasize the existence of a two-way causality between socialization decisions and political outcomes. This generates the possibility of indeterminacies and multiple self-fulfilling equilibrium paths in cultural change and politics. We provide then a rationale for ideologies and collective socialization institutions as coordination mechanisms allowing cultural groups to preserve or shift political power in favor of their preference profile in the long run. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Most formal political science and economics start from the basic assumption that individuals are endowed with exogenous and independent preferences. There is however in social sciences, from Plato to Marx and Weber, a long tradition discussing the interactions between culture and socioeconomic systems. The issue of cultural change and political institutions, in particular, has raised tremendous
interest among political scientists. Starting with the seminal work of Almond and Verba (1963), it gave birth to a large literature on political and civic culture (Almond and Verba, 1963; Eckstein, 1988; Abramson and Inglehart, 1995; Granato et al., 1996). An important conclusion from this approach has been the fact that culture and politico-economic institutions are closely related to each other and cannot be analyzed separately when taking a long enough perspective.

The purpose of this paper is to start from this insight and to make a first step into a formal analysis of the interaction between cultural change and political decisions processes. We recognize the fact that, contrary to the standard *Homo Economicus* assumption, individuals are embedded into social networks which influence, through socialization and learning, their values and views of the world (Granovetter, 1985). Economic and political decisions are then affected by the social or political culture emanating from this social matrix. Conversely however, by impacting on the allocation and evolution of resources, economic and political decisions have also implications for the evolution of the socio-cultural background in which individuals are emerged, provoking therefore generational shifts in values and cultural attitudes (Inglehart, 1990, 1997).

More precisely, we consider an economic model of socialization and cultural transmission, in which the cultural traits to be transmitted are preferences for a good whose provision is determined by a political mechanism (here voting). Parents are endowed with some paternalistic altruism with respect to their children. They do care for the future well being of their children, but can envision their offsprings’ future situation only through the filter of their own preferences. Because of this, they have a motivation to transmit their own preferences.

In this context, we emphasize the existence of a two-way causality between culture and political decision making processes. First, a profile of preferences for the publicly provided good is effective to an individual only to the extent that the political equilibrium played by his own generation reflects that profile. Hence, in any generation, parents’ gains of socialization depend on their expectation of the political aggregation of the distribution of cultural traits in their child’s population. On the other hand, the outcome of voting in any period depends on the present distribution of traits in the population, this in turn being determined by past parents’ socialization.

Given this two-way causality, we characterize the dynamics of the population distribution of preferences, and as a consequence the dynamics of political outcomes. We show the following results.

1. For unbalanced initial distributions, the dynamics display a tendency to homogeneity in the long run distribution of preferences.

The gains from socialization increase with the population share of one’s own trait. When the initial state of the population is unbalanced, one group is strongly majoritarian. Therefore, the preferences of that group are more likely to be represented in the future by the political process. This situation consequently reduces the socialization gains of the minoritarian group. It inhibits the transmis-
sion of their preferences and provides therefore a force towards long run cultural homogeneity.

(2) For relatively balanced initial distributions of preferences, the dynamics display multiple equilibrium paths generated by self-fulfilling expectations.

This result comes essentially from the two-way causality described above between shifts in political power and cultural change. When the initial state of the population of preferences is relatively balanced, the dominant group is only weakly majoritarian. In that case, it is conceivable to individuals in the minority group to reach a shift in political power in the future if enough socialization is made to affect the preferences of the next generation. Coordination of expectations on different types of future political outcomes allows then the sustainability of multiple self-fulfilling preferences paths with very different long run consequences in terms of cultural change and public policy. On the one hand, if expectations are coordinated on thinking that the minority cultural trait will be dominant in the future, there is room for an equilibrium path of preferences and political outcomes confirming these expectations and leading in the long run to homogeneity of that particular trait. On the other hand, expectations could also be coordinated to think that the minority group remains minoritarian with no shift of political power in the future. This in turn inhibits socialization to minority preferences and self-confirms the unchanged political power structure anticipated for the next period.

(3) Self-fulfilling expectations provide room for ideologies and the formation and emergence of interest groups and political entrepreneurs.

The concept of ideology is traditionally ambiguous and controversial in social sciences (Waxman, 1968; Putnam, 1971; Mullins, 1972). While various definitions have been proposed, we emphasize here the programmatic and consistency characteristics of ideologies and their role as coordination mechanisms of people’s beliefs on what should be future political outcomes and social values.

More precisely, the fact that steady state politico-cultural situations are highly dependent on self-fulfilling expectations provides a role for the programmatic and coordination aspect of ideologies as a way to select a particular path of cultural values and political power structure in society.

This role is further enlightened when cultural groups express strongly different views on political outcomes. In that case, we show that the dynamics of cultural traits and politics are characterized by important indeterminacies. The path and long run outcome actually followed by cultural change and politics depends strongly on expectations on which individuals in society coordinate. Because of different potential long run outcomes, different cultural groups may be interested into supporting different coordination mechanisms (ideologies). Hence, the possibility of conflicting ideologies.

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1 See notably Higgs (1987) and Hinich and Munger (1992) for a discussion of these various definitions.
Finally, given the externalities involved in the process of coordination of expectations and in socialization, our model of cultural transmission naturally emphasizes also a rationale for the emergence of group institutions of socialization (parties, churches, communities, lobbies, associations) designed to shift or maintain the political and cultural status quo.

Our approach is related to several literatures. First, we build on our recent work (Bisin and Verdier, 1996, 1998) on cultural transmission and socialization, which investigates the dynamics of cultural traits in a population of individuals interacting socially. Our approach extends models of population dynamics developed in Evolutionary Anthropology and Socio-biology (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985), allowing for the possibility of endogenous and costly cultural socialization and the fact that individuals do interact by making jointly political decisions.

Second, our formal study of the dynamic interactions of socialization, culture and political decision processes is, to the best of our knowledge, new. However, it has been largely studied and described informally in sociology, anthropology and political sciences (Inglehart, 1990, 1997).

Third, our analysis can be contrasted to the formal political science literature. This literature centers on comparing different forms of preference aggregation and political decision mechanisms, for given preferences. Here, however, we take the alternative point of view of taking the voting mechanism as given and to investigate the implications for the dynamics of preferences.

This paper is also obviously related to the substantial literature on political ideology (Kalt and Zupan, 1984; Higgs, 1987; North, 1990; Congleton, 1991; Hinich and Munger, 1992, 1993). This literature emphasizes the programmatic, information processing and communication aspects of ideologies and investigates the consequences on political outcomes. In common with this approach, we also consider ideologies as sets of programmatic propositions with a coordination role for expectations. Our analysis furthermore focuses on the implications of this coordination role for changes in cultural values and the feedback effects on political outcomes.

Our work is finally connected to the important literature on formation of interest groups and parties (Olson, 1965; Buchanan et al., 1980; Becker, 1983; Congleton, 1986; Austin-Smith, 1987). Compared to these approaches stressing...
mainly how institutions help to solve the free rider problem of political action and the implications on rent seeking and voting, our framework considers rather the role of political institutions as socializing devices promoting coordination of individuals’ expectations and affecting strategically the transmission and diffusion of values in society.

The paper is organized as follows. Section 2 introduces a first model of cultural transmission and majority voting between two homogeneous cultural groups. Section 3 presents results on the dynamics of culture and politics. It also discusses the rational for ideologies and collective socialization. Section 4 extends the analysis to the case of conflicting ideologies and indeterminacies in the dynamics of cultural traits, income heterogeneity and other voting mechanisms. Finally, Section 5 concludes.

2. A model of cultural transmission and voting

2.1. Preferences and voting

Consider an overlapping generation structure. In each generation, there is a continuum of agents. An individual lives for two periods, as a child and as an adult. Moreover, he has one offspring. Hence, population is stationary and normalized to one. We are concerned with the evolution of preferences for public goods or private goods publicly provided through a simple majoritarian voting mechanism. There are two possible types \(a\) and \(b\) of preferences in the population defined on a private good \(c\) and a publicly provided good \(g\). All agents want to consume the private good \(c\). However, only agents with preference \(a\) have a taste for the publicly provided good \(g\). Preferences take then the following form:

\[
\begin{align*}
    u_a(c, g) &= u(c) + \gamma u(g) \quad \text{with } \gamma > 0 \\
    u_b(c, g) &= u(c)
\end{align*}
\]

with \(u(.)\) and \(\nu(.)\) being standard strictly concave increasing functions with \(u'(0) = v'(0) = \infty\). In the beginning of their mature life, all individuals receive an identical endowment \(\hat{c}\). Provision of good \(g\) is decided in each period by majority voting of the mature generation. Though publicly provided, effective consumption of \(g\) has a small private cost of \(f\) in terms of good \(c\) (we can think of non-rival excludable publicly provided goods like museums, swimming pools or cultural activities).\(^5\) If the fraction \(q_a\) of individuals of type \(a\) is less than \(1/2\), then clearly, individuals of type \(b\) vote for \(g = 0\). On the other hand when \(q_a\) is

\(^5\) We may even think about public infrastructures which are complementary to the consumption of a second private good. In that case \(v(.g)\) is a reduced form for the consumption of that other good given a level of infrastructure \(g\).
larger than $1/2$, the voting equilibrium level of $g$, is given by the solution of the maximization program of a representative agent of type $a$:

$$\max_g u(\sigma - g - f) + \gamma v(g)$$

giving the preferred level of public good $g, (\sigma - f, \gamma)$. We will always look at the limit for $f \to 0$. Hence, it comes in the limit from $\gamma > 0$ and optimality of $g$ that $V(\sigma - f) = u(\sigma - g, (\sigma - f, \gamma) - f) + \gamma v(g, (\sigma - f, \gamma)) > u(\sigma)$ so that it is always in the interest of a mature agent of type $a$ to enjoy the good publicly provided.

2.2. Cultural transmission and socialization

Before studying in detail the interactions between voting and cultural transmission, it is useful to introduce more precisely our approach to the problem of transmission and diffusion of preferences and cultural traits (Bisin and Verdier, 1996).

We model the transmission of cultural traits and preferences as occurring through social learning. Children are born ‘naive’, i.e., with not-well-defined preferences and cultural traits. They acquire preferences through observation, imitation and adoption of cultural models with which they are matched. In particular, children are first matched with their family (‘vertical transmission’), and then with the population at large, e.g., teachers, role models, etc. (‘oblique transmission’). We also identify socialization as an economic choice (mostly of parents). In other words, parents purposefully attempt at socializing their children to a particular trait.

The motivation for a parent to socialize his child (even though socialization is costly) comes from the fact that each parent is altruistic. But, we assume, parents can perceive the welfare of their children only through the filter of their own (the parents’) preferences. This particular form of myopia (which we call ‘imperfect empathy’) is quite crucial in the analysis. In the set-up of this paper, it has the important implication that parents always want to socialize their children to their own preferences and cultural traits (because children with preferences and cultural

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6 Both the terminology of ‘vertical’ and ‘oblique’ transmission and the transmission mechanism itself are consistent with the literature in ‘cultural anthropology’: cf., e.g., Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985), and the literatures in socio-psychology and child development, cf., e.g., Bandura and Walters (1963) and Baumrind (1967). The analysis of socialization as an economic choice is in line with the literature on endogenous preferences; cf., e.g., Becker (1996).

7 While socialization may occur as the unintended by-product of some economic activity, cf., e.g., Coleman (1990), socialization is often in fact the result of resources invested purposefully by individuals and institutions: parents devote energy and money choosing the type of school and social environment in which to put their children; voters are ready to pay taxes for specific institutions preserving their cultural identities; governments allocate significant funds into programs promoting socialization to certain types of social behavior.
traits different from their parents’ would choose actions that do maximize their own and not their parents’ preferences.\footnote{This is not true in general, but only in the case of ‘pure cultural traits’, which do not affect the real side of the economy. If one particular trait for instance enlarges substantially the economic opportunities of the children, the parents might want to socialize them to this particular trait even if different from their own. Indeed, an important aspect of the present paper is to show that, in a context of public provision of certain goods through a voting mechanism, the incentives to socialize a kid to one’s own preferences depends on the expectations of the future political equilibrium.}

While a direct empirical analysis of cultural transmission mechanisms has never been pursued to the best of our knowledge, ‘imperfect empathy’ is consistent with a number of sociological and ethnographic evidence (see Bisin and Verdier, 1998 for discussion of this literature for the transmission of ethnic and religious traits).

The socialization of a naive individual occurs in two steps. First, the naive child is exposed to the parent model (type \(a\) or \(b\)) and adopts his parents’ preferences with a certain probability \(\tau^i\), \(i \in \{a,b\}\). With probability \(1 - \tau^i\), the child is matched randomly with an individual of the old generation and adopts then the preferences of that individual.

More precisely, denote \(q_t\) the fraction at time \(t\) of individuals of the old generation, which are of type \(a\). Transition probabilities \(P_{ij}^t\) that a parent of type \(i\) has a child adopting a preference of type \(j\) are then given by:

\[
\begin{align*}
P_{aa}^t &= \tau^a + (1 - \tau^a)q_t, \quad P_{ab}^t = (1 - \tau^a)(1 - q_t), \\
P_{bb}^t &= \tau^b + (1 - \tau^b)(1 - q_t), \quad P_{ba}^t = (1 - \tau^b)q_t
\end{align*}
\]

Given the transition probabilities \(P_{ij}^t\), the fraction \(q_{t+1}\) of adult individuals of type \(a\) in period \(t+1\) is easily calculated to be:

\[
q_{t+1} = q_t + \tau^a q_t (1 - q_t) [\tau^a - \tau^b]
\]

2.3. The socialization problem

There are many dimensions along which it is costly for parents to socialize their children to a certain preference pattern. Here, we simple denote with \(H(\tau^i)\) the cost of socialization effort \(\tau^i\). We assume it is twice continuously differentiable, strictly increasing and strictly convex.\footnote{Note that \(H(\tau^i)\) must be convex enough so that the solution of the socialization problem is \(\tau^i < 1\).} We assume also that \(H(0) = 0\) and \(d/d\tau^i H(0) = 0\). Formally, each parent with preferences of type \(i \in \{a,b\}\) at time \(t\) chooses \(\tau^i\) to maximize

\[
\beta \left[ P_{ii}^t V^{ii}(q_{t+1}^i) + P_{ij}^t V^{ij}(q_{t+1}^j) \right] - H(\tau^i)
\]

where \(\beta\) is the discount rate; \(P_{ii}^t\) and \(P_{ij}^t\) are the transition probabilities of the parent’s cultural trait to the child (which, as defined above in Eqs. 1 and 2), depend on \(\tau^i\) and \(q_t\); \(V^{ii}(q_{t+1}^i)\) (resp. \(V^{ij}(q_{t+1}^j)\)) denotes the utility from the economic action of a child of type \(i\) (resp. \(j\)) as perceived by a parent of type \(i\).
when he expects a future political equilibrium associated with a state of the population \( q_{t+1}^e \).\(^{10}\) More precisely,

\[
V^{au}(q_{t+1}^e) = \begin{cases} 
V(\sigma - f) & \text{if } q_{t+1}^e \geq \frac{1}{2} \\
u(\sigma) & \text{if } q_{t+1}^e < \frac{1}{2}
\end{cases}
\] (5)

\[
V^{ab}(q_{t+1}^e) = V^{ba}(q_{t+1}^e) = \begin{cases} 
\nu(\sigma - g(\sigma - f, \gamma)) & \text{if } q_{t+1}^e \geq \frac{1}{2} \\
u(\sigma) & \text{if } q_{t+1}^e < \frac{1}{2}
\end{cases}
\] (6)

\[
V^{ba}(q_{t+1}^e) = \begin{cases} 
\nu(\sigma - g(\sigma - f, \gamma) - f) & \text{if } q_{t+1}^e \geq \frac{1}{2} \\
u(\sigma) & \text{if } q_{t+1}^e < \frac{1}{2}
\end{cases}
\] (7)

Note that this definition of \( V^{ij}(q_{t+1}^e) \), for \( i,j \in \{a,b\} \), embodies ‘imperfect empathy’. In each case, a parent of type \( i \) evaluates the well being of his child only through the filter of his own preferences. The meaning of Eqs. 5–7 can be explained in a straightforward manner.

Take for instance the case of a parent of type \( a \). Consider first the case where he has a child of type \( a \). Clearly, the child’s utility is the same as the one perceived by the parent, as both individuals share the same preferences. When the public good \( g \) is provided (i.e., when \( q_{t+1}^e \geq 1/2 \), the child actually enjoys \( g \) and his utility is \( V^{au}(q_{t+1}^e) = V(\sigma - f) \). When the public good is not provided, (i.e., when \( q_{t+1}^e < 1/2 \), then the child does not consume \( g \) and gets a utility level \( V^{au}(q_{t+1}^e) = \nu(\sigma) \).

Consider now the case where the parent of type \( a \) has a child of type \( b \). When \( q_{t+1}^e \geq 1/2 \), though the public good \( g \) is provided and that taxes are raised to finance it, the child of type \( b \) does not consume it. Hence, in that case, the parent of type \( a \) perceives a child’s utility level of \( V^{ab}(q_{t+1}^e) = u(\sigma - g(\sigma - f, \gamma)) + \gamma(0) = u(\sigma - g(\sigma - f, \gamma)) \). When the public good is not provided (i.e., when \( q_{t+1}^e < 1/2 \), this utility level becomes \( V^{ab}(q_{t+1}^e) = u(\sigma) + \gamma(0) = u(\sigma) \).

Hence, individuals of type \( a \) appreciate to see their kids enjoying the public good only if the latter is actually provided. It is also clear that \( V^{au}(q_{t+1}^e) \geq V^{ab}(q_{t+1}^e) \) and individuals of type \( a \) want to socialize their children to their own preferences\(^{11}\) whenever \( q_{t+1}^e > 1/2 \) (that is, if they expect the good \( g \) to be voted by the next generation).

Parents of type \( b \) also are interested into socializing their kids to their own preferences only when good \( g \) is voted. This is so because of the cost \( f \) of using

\(^{10}\) We can separate the socio-economic and the socialization problem because we assume socialization cost enter separately into preferences. This is just for simplicity.

\(^{11}\) In the socialization problem then, without loss of generality, we allow each agent \( i \) to socialize children only to trait \( i \).
that good. As for the other group, we also have \( V^{bb}(q_{t+1}^b) \geq V^{be}(q_{t+1}^e) \). However, if the cost \( f \) is small enough, something we assume in the sequel, then individuals of type \( b \) are nearly indifferent to the type of preferences their children acquire. Hence, they do not have incentive to socialize their children to any pattern.

3. Results

3.1. Equilibrium preferences dynamics

Let \( \tau^a(q_t, q_{t+1}^a) \) and \( \tau^b(q_t, q_{t+1}^e) \) denote the solution of the socialization problem respectively for agents \( a \) and \( b \) given that the good is provided at time \( t+1 \) if and only if \( q_{t+1}^a \geq 1/2 \). Noting \( \Delta V^a = \gamma \theta(g(x_t-f)) \), direct vertical socialization for each trait is given by:

\[
\begin{align*}
\tau^a &= \begin{cases} 
H' \left( \tau^a \right) = \beta (1 - q_t) \Delta V^a & \text{if } q_{t+1}^a \geq 1/2 \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]

Note that \( \tau^a(q_t, q_{t+1}^a) \) is increasing in \( q_{t+1}^a \) as parents’ socialization gains are increasing in \( q_{t+1}^a \). At the same time, \( \tau^a(q_t, q_{t+1}^e) \) is decreasing in \( q_t \). This reflects the substitutability in socialization between family models and external models. An individual of group \( a \) has less incentives to socialize directly his child to his own trait when the fraction of that group increases because he realizes that, if not socialized by the family, the child is more likely to be socialized by an external model with trait \( a \).

Because transmitting preferences only matters if good \( g \) is provided, socialization by individuals of type \( a \) depends on expectations about the voting outcome of the next period. Preferences dynamics can be written as:

\[
q_{t+1} - q_t = \begin{cases} 
q_t (1 - q_t) H^{-1} \left[ (1 - q_t) \beta \Delta V^a \right] & \text{if } q_{t+1}^e \geq 1/2 \\
0 & \text{otherwise}
\end{cases}
\]

We can then characterize the perfect foresight path \( \{q_t\} \) of preferences in this system (i.e., \( q_{t+1}^e = q_{t+1}^e \)) as well as the equilibrium level of good \( g_t \) provided.
publicly, starting from an initial fraction \( q_0 \) of individuals of type \( a \) (the phase diagram is represented in Fig. 1).

**Proposition 1.** Suppose that for all \( x \), \( H^w(x) > \beta/4 \Delta V^a \). Then there exists a unique \( \tilde{q} < 1/2 \) such that: (1) if \( q_0 < \tilde{q} \) then for all \( t \), \((q_t, g_t) = (q_0, 0)\); (2) if \( \tilde{q} \leq q_0 < 1/2 \), then, for all \( t \), either \((q_t, g_t) = (q_0, 0)\) or \( q_t \) converges monotonically to 1 and \( g_t = g(\tilde{q}) \); (3) if \( q_0 \geq 1/2 \), then \( q_t \) converges monotonically to 1 and \( g_t = g(\tilde{q}) \).

When type \( a \) individuals are in a small enough minority, whatever their socialization effort, they will always remain a minority in the next generation. Therefore, good \( g \) will not be provided and it is not worth socializing offsprings. As nobody tries to socialize his own child, the population of preferences is entirely determined by random matching with external models and therefore remains constant. This is the intuition of (1). On the other hand in (3), preferences of type \( a \) are majoritarian in the population. As type \( b \) individuals do not socialize their children, preferences of type \( a \) can only weakly grow in the future. This implies that a majority of type \( a \) is always ensured in the next generation. This, in turn, implies that parents of type \( a \) have incentives to socialize their children to their own preferences (as the public good will be indeed provided). The number of agents of type \( a \) then increases over time and converges to 1.

Interestingly in (2), we have equilibria with self-fulfilling expectations. Initially, agents of type \( a \) are majoritarian. Suppose that they are pessimistic and believe that the majority in the next generation is again of type \( b \). Then the public good is not provided and there is no reason to socialize offsprings. This, in turn, implies that the fraction of mature individuals of type \( a \) has not changed in the next period and that the majority remains of type \( b \). This of course self-confirms the initial expectations. On the other hand, assume that individuals of type \( a \) are optimistic and believe the next period majority to be of type \( a \). Then, they expect good \( g \) to be publicly provided and they start to socialize their kids. When the fraction \( q_0 \) is larger than \( \tilde{q} \), this socialization effect is strong enough to shift the next period majority from \( b \) to \( a \), which again self-confirms the initial expectations. Once type \( a \) preferences are in majority, we are back to case (3) and the population converges to an homogeneous state with preferences of type \( a \).

### 3.2. Ideology

Ideology is a rather controversial concept in social sciences. Myriads of definitions have been given. As emphasized, however by Higgs (1987) and Hinich and Munger (1993), two characteristics appear as common to all definitions: (1) the programmatic function of ideologies (that is, a collection of statements on what
should be the state of a future society) and (2) the information processing and communication role of ideologies about politics.\footnote{Downs (1957), p. 96, describes ‘ideologies’ as ‘‘verbal images of the good society and of the chief means of constructing such a society’’. La Palombara (1968) argues that ideology ‘‘involves a philosophy of history, a view of man’s present place in it, some probable lines of future development and a set of prescriptions regarding how to hasten, retard, and/or modify that developmental directions’’. Similarly Haber (1968) says ‘‘Ideology … has several elements: (1) a set of moral values, taken as absolute, (2) an outline of the ‘‘good society’’ in which those values would be realized, (3) a systematic criticism or in the case of the status quo ideology, affirmation) of the present social arrangements and an analysis of their dynamics, (4) a strategic plan of getting from the present to the future…’’.

Here, our results on the dynamics of cultural change and politics illustrate how these salient features provide a role for ideologies as coordination mechanisms on individuals’ expectations in societies. For instance, developing an ideology that preferences of type $a$ should prevail in the future ‘‘good’’ society acts as an expectations’ coordinating device of the current generation on the idea that in the future, preferences of type $a$ will be satisfied as a political outcome. This stimulates their effort to socialize the next generation to have a preference for good $g$ which, in turn makes it possible to have the realization of this outcome.

Our analysis suggests that coordination on beliefs, through ideology, may produce coordination on preferences in the long run. Obviously, the ideology has to be consistent (i.e., self-fulfilling).\footnote{This is also an important feature of ideologies (see d’Aberbach et al., 1981; Hinich and Munger, 1993).} Therefore, it can only serve as a coordination mechanism when enough people already share a preference for good $g$ (i.e., $q_0 > \hat{q}$). When $q_0 \in [\hat{q},1/2]$, the alternative ‘‘ideology’’ that ‘‘good $g$ should never be provided by the political system’’ is also self-fulfilling. In that case, the current generation of agents of type $a$ does not socialize kids to their own preferences and preferences of type $a$ never become majoritarian in the population. The status quo remains.

It is also interesting to consider simple comparative statics on the basin of attraction of the steady state $q = 1$. It is easy to see that the larger $\Delta V^a = \gamma \sigma (\sigma - f)$, the smaller the threshold level $\hat{q}$ and the easier the sustainability of a self-fulfilling equilibrium converging towards a majority of preferences of type $a$. As $\gamma \sigma (\sigma - f)$ reflects the gains for group $a$ to shift the political outcome in the future, this result illustrates the fact that the more radical the potential political gain proposed by a coordinating ideology, the more likely is this ideology consistent (in the sense of being self-fulfilling).

3.3. Group coordination and collective socialization

So far, we assumed that socialization efforts were decided at the level of each individual family. As we saw, coordination on particular expectations of the future
may generate self-fulfilling political results, triggering a dynamics of preferences towards an even bigger representation of the preference profile of the group getting the majority. Besides this, as cultural transmission involves learning from the social environment, important externalities among and between cultural groups are not necessarily internalized at the family level. This suggests therefore that there may be some scope for political entrepreneurs or “ideologists” to come and propose the implementation of coordinated socialization efforts at the group level. What will be the effect of such a coordination institution on the dynamics of preferences and political outcomes? When would such an institution emerge? To this, we turn now in the present section.

In order to capture the logic in the simplest way, consider again the structure presented in Section 2 with two homogenous cultural groups and the public provision of one good \( g \). Individuals of group \( b \) are indifferent to socialize their own kids and so \( \tau_b = 0 \).

Consider however that the socialization decision \( \tau_a \) of people of group \( a \) is not individually decided inside each family but is designed, by some collective institution (church, community club, political party or entrepreneur) in a coordinated manner. Assume also that the socialization objective of that institution is to maximize the “imperfect empathy” altruistic utility of a member of the current generation of group \( a \).

Because the decision on \( \tau_a \) is now taken at the group level, it has to take into account the impact of socialization on the dynamics of cultural traits and political outcomes. Hence, the socialization problem for group \( a \) is given by:

\[
\text{Max}_{\tau_a} \beta \left[ \tau_a + (1 - \tau_a) q_i \right] \Delta V^a(q_{t+1}) - H(\tau_a) \]

\[
u_t.\Delta V^a(q_{t+1}) = \gamma_t \left( g(\sigma_i) \right) \quad \text{if } q_{t+1} \geq \frac{1}{2}
\]

\[
= 0 \quad \text{if } q_{t+1} < \frac{1}{2}
\]

and 

\[
q_{t+1} = q_t + q_t \left( 1 - q_t \right) \tau_a
\]

We have then the following proposition:

**Proposition 2.** Suppose that \( H(.) \) is convex enough, then there exists a unique \( q^* < 1 / 2 \) such that: (1) if \( q_0 < q^* \) then for all \( t \), \( \tau^*_t = 0 \), \( q_t = q_0 \) and the political equilibrium is \( g^*_t = 0 \); (2) if \( q^* \leq q_0 \), then \( \tau^*_t > 0 \), \( q_t \) converges monotonically to 1. The political equilibrium is \( g^*_t = g(\sigma) \) for all \( t \geq 1 \). (3) The threshold under coordination \( q^* \) is smaller than the threshold without coordination \( \hat{q} \).

\[\text{14 Obviously, one could also think about a collective institution having a more forward looking objective. This will complicate the optimal socialization decisions but will not affect the main conclusion of this section.}\]
Proposition 2 says that in the case of coordination, there is a threshold level $q^c$ of the fraction of individuals of type $a$, (smaller than the initial political majority composed of group $b$), such that above that threshold level, a coordinated pattern of socialization by group $a$ will prove effective to shift the future structure of preferences in the population and to affect dramatically the pattern of future political outcomes. Moreover, this threshold is necessarily smaller than the threshold triggering a similar process when socialization is not coordinated. By coordinating their socialization efforts, individuals of group $a$ are taking into account the political externality that current socialization not only affects the paternalistic payoff of one particular individual of group $a$ for his kid but, also, is actually improving the expected welfare of all the other individuals sharing the same preference and transmitting that preference to their kid.

This proposition provides also a rationale for the existence of group institutions with the purpose of affecting in a coordinated way, the pattern of socialization across individuals. Suppose for instance that there is a fixed cost $C$ to set up an institution, which provides a coordinated socialization effort to individuals of group $a$. Denote also $\hat{\tau}^a(q)$ the level of socialization shifting in one generation group $a$ from a minority to a majority (i.e., such that $q + q(1 - q) \hat{\tau}^a = 1/2$).

Consider the group’s benefit from such a socialization effort, $qV(\hat{\tau}^a, q) = q[\tau^a + (1 - \hat{\tau}^a)q] \beta
\nu(\nu(q) - H(\hat{\tau}^a))$. It can be readily seen that $qV(\hat{\tau}^a, q)$ is increasing in $q$. From Proposition 2, when group $a$ is a small minority (i.e., $q < q^c$), there is no benefit for a coordinated positive socialization effort. Hence, no possibility for the emergence of a collective institution of preference formation. When group $a$ is not too small a minority (i.e., $q > q^c$), then the group’s benefits from collective socialization $qV(\hat{\tau}^a, q)$ becomes positive and for $q$ large enough, can possibly overcome the fixed cost, $C$. In that case, there is scope for the emergence of a political entrepreneur proposing the implementation of a coordinated effort of socialization to cultural trait $a$.

Interestingly, one should note however that when group $a$ is larger than the non-coordinated threshold $\hat{q}$, the benefit to have a political entrepreneur is not anymore ensured. Indeed in that case, individual socialization efforts at the family level are enough to set the process of cultural change if expectations are well coordinated by the existence of a well-defined ideology. Obviously, such an ideology could be distilled by a political entrepreneur but it need not be so. This suggests, therefore, a non-monotonicity in the role of political entrepreneurs for cultural change. Political entrepreneurs are more likely to propose coordinated socialization efforts whenever the cultural trait they support is of an intermediate minority size.

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15 We thank an anonymous referee for bringing this aspect to our attention.
16 For instance, the role of “benevolent” intellectuals and the diffusion of their ideas and conceptions may be an alternative channel to the formation of ideologies in societies.
4. Extensions

4.1. Conflicting ideologies

Until now, we assumed that group $b$ was passive in its behavior of socialization (i.e., $\tau^b = 0$). When this is not the case, our previous framework applied symmetrically to both groups ($a$ and $b$) suggests that there is a range of initial conditions under which, expectations about future political outcomes will play an even bigger role to determine the trajectory of preferences and voting. Actually, for such a range of initial conditions, there may be multiple equilibrium paths $\{q_t\}$, some of which, converging in the long run to $q = 0$, or on the contrary to $q = 1$.\footnote{As in Krugman (1991) and Matsuyama (1991) but in a different context. History and expectations can be strongly intertwined in the evolution of social systems. While Krugman and Matsuyama were interested in economic growth and intersectoral reallocation of resources, here we are emphasizing a similar point for the evolution of preferences on goods decided through public decision making processes.}

To see this in the simplest way, consider again our initial model of Section 2 but now with two types of publicly provided goods $g_a$ and $g_b$. We assume that group $i$ ($i = a, b$) has a preference structure of the following type

$$u_i(c, g_i, g_j) = u(c) + \gamma_i u'(g_i) + \gamma_j u'(g_j) \quad \text{with } \gamma_i > 0 \text{ and } \gamma_j = 0$$

with $u(.)$ and $v(.)$ being standard strictly concave increasing functions with $u(0) = v(0) = 0$ and $u'(0) = v'(0) = \infty$. Hence, group $a$ (resp. $b$) has a preference for good $g_a$ (resp. $g_b$). Again, in the beginning of their mature life, all individuals receive an identical endowment $\sigma$. Provided that, effective consumption of $g_i$ has a (small) private cost of $f$ in terms of good $c$, it is clear that group $i$ will consume its own preferred good $g_i$ when it is publicly provided and will not consume the other good. In each period, it is also clear that individuals of group $i$ vote for the provision of good $i$. Hence, when the fraction $q_i$ of individuals of type $a$ is less than $1/2$ (resp. larger than $1/2$), the political equilibrium provides $g_{ai} = 0$ and $g_{bi} = \cdot \cdot \cdot g_{bi}^*$, (res. $g_{ai} = g_{ai}^*$ and $g_{bi} = 0$) with $g_{ai}^*$ and $g_{bi}^*$ the preferred level of good of group $a$ and $b$ (i.e., the solutions of:

$$\text{Max}_g \quad u(\sigma - g_i - f) + \gamma_i u'(g_i) \quad \text{for } i = a, b.$$  

Note that there is the usual problem, when $q_i = 1/2$, to determine which group wins the elections. In order to avoid this, we simply assume that to have a clear-cut public decision, any group needs to have a majority by a small margin $\epsilon$ in order to be able to implement its preferred policy. In the sequel, we let $\epsilon$ be arbitrarily small ($\epsilon \to 0$).
Denote $V_a(\sigma) = \gamma_a, \beta(g_a^*)$. Then the perceived gains of socialization for each group can be readily computed as (when $f \to 0$):

$$
\Delta V^u(q_{t+1}^e) = \begin{cases} 
V_a(\sigma) & \text{if } q_{t+1}^e \geq \frac{1}{2} + \epsilon \\
0 & \text{if } q_{t+1}^e < \frac{1}{2} + \epsilon
\end{cases}
$$

and:

$$
\Delta V^b(q_{t+1}^e) = \begin{cases} 
0 & \text{if } q_{t+1}^e > \frac{1}{2} - \epsilon \\
V_b(\sigma) & \text{if } q_{t+1}^e \leq \frac{1}{2} - \epsilon
\end{cases}
$$

From this, direct vertical socialization for each trait is given straightforwardly as:

$$
H^u(\tau^u) = \beta (1 - q_t) V_a(\sigma) \quad \text{if } q_{t+1}^e \geq 1/2 + \epsilon
$$

$$
\tau^u = 0 \quad \text{otherwise}
$$

and:

$$
H^b(\tau^b) = \beta q_t V_b(\sigma) \quad \text{if } q_{t+1}^e \leq 1/2 - \epsilon
$$

$$
\tau^b = 0 \quad \text{otherwise}
$$

Preferences dynamics are written as:

$$
q_{t+1}^a - q_t = \begin{cases} 
q_t (1 - q_t) H^{-1} (1 - q_t) \beta V_a(\sigma) & \text{if } q_{t+1}^e \geq 1/2 + \epsilon \\
-q_t (1 - q_t) H^{-1} (q_t \beta V_b(\sigma)) & \text{if } q_{t+1}^e \leq 1/2 - \epsilon \\
0 & \text{else}
\end{cases}
$$

We can then characterize the perfect foresight path $(q_t)$ of preferences in this system (i.e., $q_{t+1}^e = q_t$) starting from an initial fraction $q_0$ of individuals of type $a$ (the phase diagram is represented in Fig. 2).

**Proposition 3.** Suppose that $H(.)$ is convex enough and that $\epsilon \to 0$. Then there exists a unique $\hat{q}_a < 1/2$ and a unique $\hat{q}_b > 1/2$ such that: (1) if $q_0 < \hat{q}_a$, then $q_t$ converges monotonically to $0$; (2) if $q_0 > \hat{q}_b$, then $q_t$ converges monotonically to $1$; (3) if $\hat{q}_a \leq q_0 \leq \hat{q}_b$, then, there are multiple rational expectation paths, some of which converging to $0$, some of which converging to $1$.

The main implication of Proposition 3 is to show that when the state of the initial population of preferences is relatively balanced, then the role of expecta-
tions and their coordination on a certain future outcome become important for the
determination of the long run evolution of preferences and cultural change. It
clearly illustrates then how important ideologies reflecting different views can be a
crucial feature of political dynamics.\textsuperscript{18}

### 4.2. Cultural and social heterogeneity

So far, the only source of heterogeneity across individuals was a difference in
their preference for good $g$. The result of this is a discontinuous pattern in term of
the voting outcome at each period. Or cultural group $a$ is majoritarian and the
public good is supplied at an optimal constant rate according to the preference of
that group; or conversely, it is group $b$ which has a substantial majority and no
public good is voted in equilibrium with no dynamics of preferences. In this
section, we introduce another dimension of heterogeneity between agents in terms
of their endowments $v$ at the beginning of their mature life. This feature will
introduce interesting new political cleavages (social and cultural) and allows for a
redistributive dimension within cultural group $a$ (which has a preference for good $g$).

More precisely, assume that at the beginning of their mature life, all individuals
face an idiosyncratic shock, on their productivity say, such that their endowment
$\omega$ is distributed according to a distribution with a cumulative function $F(\omega)$ on a
compact support $[\omega_{\min}, \omega_{\max}]$, with a mean value $\mu$. After the realization of this

\textsuperscript{18} It is not difficult to see that with an initially relatively well-balanced population of the two cultural
traits (i.e., $q_a \leq q_b \leq q_a$), one can also generate cycles in preferences and politics associated to a
particular pattern of expectations on future outcomes.
shock, individuals vote on the provision of good \( g \) financed by a flat tax rate \( \theta \) on income \( \omega \) of all individuals (type \( a \) and \( b \)) of the current mature generation.\(^9\)

A time \( t \), in a mature generation, the voting game is solved by first looking at the preferred policy of an individual of type \((i, \omega)\). Clearly, individuals of type \( b \) do not vote for the provision of the public good. Hence, for them \( \theta_b = 0 \). An individual of type \((a, \omega)\) decides his preferred policy according to the following program:

\[
\text{Max}_\theta u(\omega(1 - \theta) - f) + \gamma v(\theta \sigma)
\]

where we make use of the government budget constraint \( g = \theta \sigma \). The first order condition gives:

\[
\omega u'(\omega(1 - \theta) - f) = \gamma \sigma v'(\theta \sigma)
\]

leading to an optimal tax rate \( \theta^*(\omega, \sigma) \), which, given the concavity assumptions on \( u(.) \) and \( v(.) \) satisfies the second order condition. Because of the single peakness property of the utility function of all agents in this economy, the political equilibrium of this voting game is the preferred tax rate of the median of the distribution of voters. This median is given by: an agent of type \( b \) when \( q_i < 1/2 \), and an agent of type \( a \) with an endowment \( \sigma^m(q_i) \) such that

\[
F(\sigma^m(q_i)) = \frac{1}{2q_i} \quad \text{when } q_i \geq \frac{1}{2}
\]

Consequently, the voting equilibrium tax rate and level of public good is given by:

\[
\theta^* = 0 \text{ and } g^* = 0 \quad \text{when } q_i < \frac{1}{2}
\]

\[
\theta^* = \theta^*(q_i) = \theta_a(\sigma^m(q_i), \sigma) \quad \text{when } q_i \geq \frac{1}{2}
\]

and

\[
g^* = g^*(q_i) = \sigma \theta_a(\sigma^m(q_i), \sigma)
\]

Consider then the following assumption:

Assumption A: for all \( x \), \( xu''(x) + u'(x) < 0 \) and \( xv''(x) + v'(x) < 0 \).

Assumption A ensures that, quite intuitively, \( \theta_a(\omega, \sigma) \) is decreasing in \( \omega \) and increasing in \( \sigma \) (i.e., poor individuals want more taxes and public provision of good \( g \)). Hence, taking into account (8), \( g^*(q_i) \) is increasing in \( q_i \), the fraction of individuals of type \( a \).

---

\(^9\) We assume that the rate of transformation from taxes \( \theta \sigma \) to \( g \) is equal to 1 for simplicity so that \( g = \theta \sigma \).
To study cultural transmission in such a setting, we need to compute the "perceived" expected gain for an individual of type \( i \) to have a child of the same type. First we have for \( f^0 = 0 \):

\[
V^{aa}(q^*_{i+1}) = E_a u\left[ \omega (1 - \theta^a(q^*_{i+1})) \right] + \gamma \nu (\theta^a(q^*_{i+1}) \sigma)
\]  

(9)

and:

\[
V^{ab}(q^*_{i+1}) = V^{bb}(q^*_{i+1}) = V^{ba}(q^*_{i+1}) = E_a u\left[ \omega (1 - \theta^a(q^*_{i+1})) \right]
\]  

(10)

Note now that parents need to take into account expectations about their child’s future endowment \( \omega \). Again, as in Section 4.1, the political equilibrium tax rate \( \theta^a(q^*_{i+1}) \) is dependent on the future fraction of individuals \( q^*_{i+1} \) of type \( a \). Hence, the "perceived" child’s utility is also dependent on this future fraction. Using Eqs. 9 and 10 and noting now \( \Delta V^a(q^*_{i+1}) = \gamma \nu (\theta^a(q^*_{i+1}) \sigma) \), one gets the following dynamics for cultural transmission:

\[
q_{i+1} - q_i = \begin{cases} 
q_i (1 - q_i) H^{-1} \left[ (1 - q_i) \beta \Delta V^a(q^*_{i+1}) \right] & \text{if } q^*_{i+1} \geq 1/2 \\
0 & \text{else}
\end{cases}
\]

The difference between the present situation and Section 4.1 is the fact that when the future generation of individuals of type \( a \) is a majority, the gain to socialization to trait \( a \), \( \Delta V^a(q^*_{i+1}) \), is not anymore a constant but depends now on the expected value \( q^*_{i+1} \). Perfect foresight equilibrium paths \( \{q_i\} \) can be then characterized in the following proposition:

**Proposition 4.** Suppose assumption A holds and that \( H(s) \) is sufficiently convex. Then there exists a unique \( \tilde{q} < 1/2 \) such that: (1) if \( q_0 < \tilde{q} \) then for all \( t \), \( q_t = q_0 \); (2) if \( \tilde{q} \leq q_0 < 1/2 \), then, for all \( t \), either \( q_t = q_0 \) or \( q_t \) converges monotonically to \( 1 \); (3) if \( q_0 \geq 1/2 \), then \( q_t \) converges monotonically to \( 1 \). Whenever \( q_t \) converges monotonically to \( 1 \), taxes and public good provision increase monotonically from \( \theta^a(q_t) \) (resp. \( \theta^a(q_t) \sigma \)) to \( \theta^a(1) \) (resp. \( \theta^a(1) \sigma \)).

The intuition behind Proposition 4 is the same as in Proposition 1. When the initial fraction of individuals of type \( a \) is small enough, there is no change of political majority. However, when group \( a \) is a big enough minority, there is a possibility to trigger a process of cultural change converging in the long run towards homogeneity \( q = 1 \). Proposition 4 adds up non-trivial dynamics of taxation and public provision of good \( g \). Indeed, as soon as group \( a \) becomes a political majority, taxes and provision of \( g \) increase. Initially the decisive agent is not the median \( \omega^a \) of the distribution of \( F(\omega) \), as there is a "cross cultural" coalition of agents of type \( b \) and rich agents of type \( a \) who oppose the platform preferred by the median \( \omega^a \). However, when trait \( a \) becomes majoritarian in the population, the political median comes closer to the median of the distribution of
endowments and, in the end, taxes and provision of $g$ converge to the preferred outcome of the median voter $\omega$.

4.3. Voting mechanisms

In the basic model, we considered the simplest voting mechanism: direct majority with no uncertainty. It is immediate to see however that most of the results of the basic model are robust to more general voting mechanisms. Indeed, with a weighted voting mechanism where one group is more heavily weighted in the political game, it is easy to see that the results of Proposition 1 remain qualitatively valid. The threshold $\hat{q}$ above which one may get an equilibrium path $(q_t)_{t \geq 0}$ converging to a corner point, is now simply affected quantitatively by the voting weight structure.

Similarly, we assumed that people had complete knowledge on the structure of preferences at each point in time. Hence, they could fully anticipate the position of the politically decisive agent and the threshold which had to be passed to get a shift of power in the future. This, of course, need not be so and the outcome of future political equilibria could be actually a probabilistic function of the fraction of individuals sharing the same preferences. What matters though, in order to have our results qualitatively extended is the fact that: (1) the gains of socialization are increasing in $q$, (2) there exists some threshold in $q$ above which good $g$ can be publicly provided with a high enough probability. In that case, there will still be room for some coordination on expectations, inducing a pattern of homogenization of preferences.

5. Conclusion

In this paper, we analyzed how cultural change in a society interacts with public decision making processes. Though we considered a very rudimentary model of politics (a simple direct majority voting mechanism), our framework enabled us to discuss how political institutions affect the evolution of cultural traits. At the same time, socialization efforts spent by parents and collective institutions determine the profiles of views existing in future generations and have clearly implications for the design of future public decisions. This two-way causality generated an important role for expectations about future political outcomes, emphasizing therefore the role of ideologies as coordination mechanisms of expectations. Our framework also allowed us discussing a rationale for collective institutions of socialization as a strategic way to preserve or shift political power, through manipulation of the diffusion of values in the society.

Our work has to be seen as a first step at integrating in a formalized way, the interactions between cultural change and politico-economic institutions. Clearly, much remains to be done in this field. An important aspect left aside is the role of
efficiency constraints on the dynamics of political cultures and ideologies. In the present paper, the two types of political preferences generated equally efficient ‘‘economic relationships’’. Therefore, our analysis focused on ‘‘pure’’ cultural transmission implications of ideologies. Obviously, from past history, we know that relative economic success also plays an important role as a selective force on types of economic organizations and the underlying cultural structure congruent with them. A natural extension is to integrate ‘‘efficiency’’ and ‘‘socialization’’ selective mechanisms into an unified framework. While this is certainly beyond the scope of this paper, we hope however that the framework developed here can serve as a useful building block for future research along these lines.

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Appendix A

Proposition 1: Define

\[ A(q) = q + q(1 - q) \Delta H^{-i}[(1 - q) \beta V^a] \]

Immediately, \( A(q) \geq q \geq 1/2 \) (with = only if \( q = 1 \)). This proves (3). Then define \( \hat{q} \) to solve \( A(\hat{q}) = 1/2 \). Since:

\[ A'(q) = 1 + (1 - 2q) \Delta H^{-i}[(1 - q) \beta V^a] - \frac{q(1 - q) \beta V^a}{\Delta H^i \Delta V^a} \]

As by assumption:

\[ H^i(\hat{x}) > \frac{\beta}{4} \Delta V^a \]

See notably Ursprung (1988) for a discussion of related issues between cultural transmission and efficiency selection forces.
we can rewrite
\[
A'(q) = (1 - 2q) H^{-1}[(1 - q) \beta \Delta V^a] + 1 - \frac{q(1 - q) \beta \Delta V^a}{H' \circ H'^{-1}[(1 - q) \beta \Delta V^a]}
\]
\[
> (1 - 2q) H^{-1}[(1 - q) \beta \Delta V^a] \\
+ 1 - \frac{\beta \Delta V^a}{4[H' \circ H'^{-1}[(1 - q) \beta \Delta V^a]]}
\]
\[
> (1 - 2q) H^{-1}[(1 - q) \beta \Delta V^a]
\]
Hence, $A'(q) > 0$ for $q < 1/2$. Moreover $A(0) = 0$, and $A(1/2) > 1/2$, thus there exists a unique $\hat{q} < 1/2$ such that $A(\hat{q}) = 1/2$. Moreover, $q < \hat{q}$ implies $A(q) < 1/2$. This proves (1) and (2).

Proposition 2: It is clear that for $q_0 \geq 1/2$, the political equilibrium is $q^*_a = g(\sigma_a)$, that $\tau^a > 0$ and that $q_a$ converges monotonically to 1. Consider now the case $q_0 < 1/2$. As long as $q_{t+1} = q_t(1 - q_t)\tau^a < 1/2$, there are no incentives for group $a$ to socialize its kids, as the next period political equilibrium is still with a group $b$ majority. Therefore, when the fraction of individuals of type $a$ is $q$, group $a$ has to undertake a minimum level

\[
\tilde{\tau}^a = \frac{1/2 - q}{q(1 - q)}
\]

to make sure that in the next period, the next majority is in group $a$. Compared to the alternative of doing nothing $\tau^a = 0$, it is profitable to undertake $\tilde{\tau}^a$ in a coordinated fashion when:

\[
\beta [\tilde{\tau}^a + (1 - \tilde{\tau}^a) q] \gamma(\sigma_a) - H(\tilde{\tau}^a) > 0
\]

In order to investigate this issue, denote by $\tilde{\tau}^a(q)$ the value of $\tau$ different from 0 such that $V(\tau, q) = \beta[\tau + (1 - \tau) q] \gamma(\sigma_a) - H(\tau) = 0$. It is easy to see that $V(\tau, q)$ has a positive optimum at $\tau = H^{-1}[\beta(1 - q) \gamma(\sigma_a)]$. Hence, $\tilde{\tau}^a(q) > H^{-1}[\beta(1 - q) \gamma(\sigma_a)]$ and $V(\tau, q) \geq 0$ as long as $\tau \leq \tilde{\tau}^a(q)$. Moreover, by differentiation, $\tilde{\tau}^a(q)$ is increasing in $q$ from $\tilde{\tau}^a(0) > 0$ to $\tilde{\tau}^a(1) < 1$. At the same time, differentiation of $\tilde{\tau}^a(q)$ provides that $\tilde{\tau}^a(q)$ is decreasing in $q$ when $q \in [0, 1/2]$ from $+\infty$ to 0. Hence, there is a unique $q^* < 1/2$ such that $\tilde{\tau}^a(q^*) = \tilde{\tau}^a(q^*)$.

For $q < q^*$, one has $\tilde{\tau}^a(q) > \tilde{\tau}^a(q^*)$, hence, $V(\tilde{\tau}^a(q), q) < 0$ and it is profitable to group $a$ to socialize its kids to the point that there is a future shift in majority in the political equilibrium. Hence, $\tau^a = 0$, $q_{t+1} = q_t < 1/2$ and there is no public provision of $g$. When $q \geq q^*$, then $\tilde{\tau}^a(q) > \tilde{\tau}^a(q^*)$, hence, $V(\tilde{\tau}^a(q), q) > 0$. It is profitable to group $a$ to socialize its kids to the point that there is a future shift in majority. Hence, $\tau^a = \tilde{\tau}^a(q^*) > 0$, $q_{t+1} \geq 1/2$ and there is public provision of future periods. (1) and (2) follow immediately.
From Proposition 1, \( \hat{q} \) is determined by:

\[
\hat{q} + \hat{q}(1 - \hat{q}) H^{-1} \left[ \beta(1 - \hat{q}) \gamma \nu(g(\sigma)) \right] = 1/2
\]

Also when \( H(.) \) is convex enough, the function \( A(q) = q + q(1 - q)H^{-1}[ \beta(1 - q)\gamma \nu(g(\sigma))] \) is increasing in \( q \) for \( q \in [0, 1/2] \). \( \tilde{\tau}^a(q) \) is defined by:

\[
q + q(1 - q)\tilde{\tau}^a(q) = \frac{1}{2}
\]

Assume now that \( \hat{q} < q^c \). Then

\[
A(\hat{q}) = 1/2 < A(q^c) = q^c + q^c(1 - q^c) H^{-1} \left[ \beta(1 - q^c) \gamma \nu(g(\sigma)) \right]
\]

But at \( q^c \), one has \( q^c + q^c(1 - q^c)\tilde{\tau}^a(q^c) = 1/2 = q^c + q^c(1 - q^c)\tau^0(q^c) \). Hence, it follows that \( \tau^a(q^c) < H^{-1}[ \beta(1 - q^c)\gamma \nu(g(\sigma))] \), which contradicts the fact that for all \( q \in [0, 1] \), \( \tau^a(q) > H^{-1}(\beta \tau + (1 - \tau)q) \gamma \nu(g(\sigma)) \). Hence (3) follows by contradiction.

Proposition 3: Define, \( A(q) \) as in the proof of Proposition 1 with \( \Delta V^a = V_a(\sigma) \) and in a symmetric way:

\[
B(q) = q - q(1 - q) H^{-1}[q \beta \Delta V^b]
\]

with \( \Delta V^b = V_b(\sigma) \). The dynamics of preferences can be written as:

\[
q_{t+1} = \begin{cases} 
A(q_t) & \text{if } q_{t+1}^c \geq 1/2 + \varepsilon \\
-B(q_t) & \text{if } q_{t+1}^c \leq 1/2 - \varepsilon \\
0 & \text{else}
\end{cases}
\]

One sees immediately that \( B(q) \leq q \) for \( q \leq 1/2 \) (with = only if \( q = 0 \)). Also:

\[
B'(q) = 1 - (1 - 2q)H^{-1}[q \beta \Delta V^b] - \frac{q(1 - q) \beta \Delta V^b}{H^a \circ H^{-1}[q \beta \Delta V^b]}
\]

When \( H(.) \) is convex enough, an argument symmetric to the one of the proof of Proposition 1 gives that \( B'(q) > 0 \) for \( q > 1/2 \). Moreover \( B(1) = 1 \), and \( B(1/2) < 1/2 \), thus, using the arguments of Proposition 1, there exists a unique \( \hat{q}_a(\varepsilon) \) and a unique \( \hat{q}_b(\varepsilon) \) for \( \varepsilon \) small enough. Taking the limit of \( \varepsilon \to 0 \), we get \( \hat{q}_a \) and \( \hat{q}_b \) as announced in Proposition 2. Also, \( q < \hat{q}_a \) implies \( A(q) < 1/2 \) and \( q > \hat{q}_b \) implies \( B(q) > 1/2 \). This proves (1) and (2). (3) comes from the fact that when \( \hat{q}_a < q < \hat{q}_b \), the dynamics depends on the individuals’ expectation \( q_{t+1}^c \) being larger or smaller than \( 1/2 \) at a certain point of time \( t \). By definition of \( \hat{q}_a \) and \( \hat{q}_b \), both types of expectations are actually consistent with a rational expectation equilibrium.
Proposition 4: Define again $A(q,q') = q + q(1-q)H^{-1}[(1-q)\beta \Delta V^{*}(q')]$. Immediately, $A(q)\geq q$ for $q\geq 1/2$ (with = only if $q = 1$). This proves (iii). Then consider $\tilde{q}$ such that $A(\tilde{q},1/2) = 1/2$. Since:

$$A'(q,1/2) = 1 + (1-2q)H^{-1}[(1-q)\beta \Delta V^{*}(1/2)] - \frac{q(1-q)\beta \Delta V^{*}(1/2)}{H^* \circ H^{-1}[(1-q)\beta \Delta V^{*}(1/2)]}$$

When $H(x)$ is convex enough, it is again easy to see that $A'(q,1/2) > 0$ for $q < 1/2$. Moreover $A(0) = 0$, and $A(1/2) > 1/2$, thus there exists a unique $\tilde{q} < 1/2$ such that $A(q,1/2) = 1/2$. As $A(q,q')$ is increasing in $q'$, one has necessarily for any $q \geq \tilde{q}$ and $q' > 1/2$, $A(q,q') > A(q,1/2) = A(\tilde{q},1/2) > 1/2$. This proves (ii). For (i), the proof is by contradiction. Assume that there is a rational expectation path $\{q_t\}$ starting from a point $q_0 < \tilde{q}$ and such that the dynamics of $q_t$ is implicitly described by $q_{t+1} = A(q_t, q_{t+1})$. This can be the case only if $q_{t+1} \geq 1/2$. In particular, $q_1 > 1/2$. Differentiation of $q_{t+1} = A(q_t, q_{t+1})$ implies:

$$\frac{dq_{t+1}}{dq_t} = \frac{A_q(q_t, q_{t+1})}{1 - A_{q_{t+1}}'(q_t, q_{t+1})}$$

When $H(x)$ is convex enough, it is easy to see that $A_q(q_t, q_{t+1}) > 0$ and $1 - A_{q_{t+1}}'(q_t, q_{t+1}) > 0$ implying under these dynamics that $q_{t+1}(q_t)$ is positively related to $q_t$. Hence, $q_t(q_{t_0}) < q_t(\tilde{q})$. But, by definition of $\tilde{q}$, $q_t(\tilde{q}) = A(\tilde{q},1/2) = 1/2$. Hence, $q_t(q_{t_0}) < 1/2$ proving (i) by contradiction.

References

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