# A General Equilibrium Theory of Capital Structure*

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**Abstract**

We develop a general equilibrium theory of the capital structures of banks and firms. The liquidity services of bank deposits make deposits a "cheaper" source of funding than equity. Banks pass on part of this funding advantage in the form of lower interest rates to firms that borrow from them. Firms and banks choose their capital structures to balance the funding of debt against the risk of costly default. Firm equity is a substitute for bank equity. An increase in a firm's equity makes the firm's debt less risky and that in turn reduces the risk of the bank's portfolio. Firms have a comparative advantage in providing a buffer against systemic shocks, whereas banks have a comparative advantage in providing a buffer against idiosyncratic shocks.

## 1 Introduction

A firm's choice of capital structures has general equilibrium repercussions. A firm that uses a bank loan to fund investment or operations may run the risk of costly default. If the firm has a large equity buffer, the risk of default will be lower and the loan will be safer. This in turn will make the bank's asset safer and reduce the bank's risk of default. This general

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effect—what Gornall and Strebulaev (2015) have called the "supply chain" of finance—raises interesting questions about the determination and optimality of capital structures in general equilibrium. In this paper, we develop a general equilibrium theory of the capital structures of banks and firms. We show that, if markets for debt and equity are complete, the equilibrium capital structures will be constrained efficient. Before equity in firms is a substitute for equity in banks, we expect bank leverage to be higher than firm leverage, but firms and banks have their own comparative advantages when it comes to providing a buffer against default. Firms have a comparative advantage in providing buffers against systemic shocks, whereas banks have a comparative advantage in providing buffers against idiosyncratic shocks.

Admati and Hellwig (2013) have argued that the seminal paper of Modigliani and Miller (1958) should be the starting point for any discussion of bank capital regulation. Modigliani and Miller (1958) was indeed the cornerstone of what became the theory of corporate capital structure, but the environment considered by Modigliani and Miller is not appropriate for studying the determinants of bank capital structure. In analyzing a bank's choice of capital structure, it is important to recognize its role as an intermediary and the fact that bank deposits are not merely debt claims, but also function as *money*. In this paper, we assume that deposits are used for transactions and this function gives rise to a spread between the return on equity and the return on deposits. At the same time, the debt-like properties of deposits give rise to the possibility of costly default.[1]

In our model, capital structure matters, unlike the Modigliani-Miller model, where capital structure is irrelevant. A bank's capital structure is determined by a trade-off between the funding advantage of deposits and the possibility of costly default[2]. In addition, a bank's choice of capital structure is affected by the firms that borrow from the bank. The higher the firms' leverage, the riskier their loans, and the higher the probability of default by the bank, other things being equal. This interdependence between leverage in the corporate sector and leverage in the banking sector is a major theme of this paper.

We study a representative agent economy consisting of consumers, firms and banks. Consumers are the initial owners of capital goods and and the ultimate buyers of consumption goods. Firms have access to a variety of risky technologies, which use capital goods as inputs to the production of consumption goods. Firms issue equity to households and borrow from banks in order to fund the purchase of capital goods. Banks lend to firms and issue equity and deposits to households to fund their loan portfolios. Households purchase equity in firms

[1] We assume that when a bank fails, because it is unable to meet the demand for withdrawals, a fraction of the value of its assets is lost.

[2] A similar trade-off arises in models of optimal capital structure where the firm balances the tax advantages of debt and the costs of default.

and make deposits in banks to fund their future consumption. Only banks can lend to firms and only households can invest in equity.

Firms and banks are restricted in the securities they can issue. Firms are restricted to issuing debt and equity and banks are restricted to taking deposits and issuing equity. In this sense, markets are incomplete. Nonetheless, the set of potential securities that can be issued is large, because firms and banks can make different choices regarding technologies (in the case of firms), loan portfolios (in the case of banks), and capital structures (in the case of both firms and banks). These choices affect the risk characteristics of the securities issued by banks and firms and result in a large array of potential securities being priced and traded. We assume markets for these potential securities are competitive and *complete*, in the sense that there is a market (and a market-clearing price) for every *possible* type of security that can be issued.

In this framework, we obtain analogues of the fundamental theorems of welfare economics. First, we show that a competitive equilibrium, where firms and banks maximize their market value, is constrained efficient.[3] Second, we show that any constrained efficient allocation can be decentralized[4] as an equilibrium.[5]

Although the welfare theorems have a familiar ring, the assumptions are demanding. In our framework, both firms and banks innovate by creating new securities. When a firm changes its capital structure or chooses a riskier technology, it produces new forms of (risky) debt and equity. Similarly, when a bank changes its capital structure and portfolio, it produces new forms of (risky) deposits and equity. The coordination of these choices depends on the market clearing prices and infinite number of securities that are not traded in equilibrium.

The welfare theorems are interesting in their own right, but they are also crucial for characterizing the equilibrium capital structures. Solving for an equilibrium is difficult, but we can use the efficiency conditions to characterize the capital structures in the banking and corporate sectors.

We next turn to the critical question: *What is the efficient allocation of equity between the banking and corporate sectors*? We begin by looking at a partial equilibrium, single factor

[3]The first best or a Pareto efficient allocation cannot be attained because of the restriction to debt and equity as the funding instruments available to firms and banks. However, if we similarly restrict the planner to the allocations attainable using debt and equity, using so a notion of constrained efficiency, he would not be able to improve on the laisser faire equilibrium.

[4]Since there is a representative consumer lump sum taxes and transfers are not needed to decentralize the constrained efficient allocation.

[5]In this paper we ignore asymmetric information in order to focus on the welfare properties of the choices made by banks and firms in a basic competitive environment. There is a large and well known literature on moral hazard and risk shifting in banks. These considerations, as well as the possibility of bank bailouts in the event of default, may distort the choice of the capital structure and introduce inefficiencies.

model. There is a large number of technologies with productivities that are the product of a systemic and an indiosyncratic shocks. Although the firms' equity will absorb the systemic shock, bank equity is needed to efficiently buffer the idiosycratic shocks. In fact, without this kind of idiosyncratic shock, it would be efficient for banks to have no equity capital in the single factor model.

Similar results hold in the general equilibrium model. If the technologies' returns satisfy a property known as *co-monotonicity*, an efficient allocation requires all of the equity capital to be in the corporate sector. To get positive bank capital there must be some idiosyncratic risk. Because of the non-convexities generated by the bankruptcy costs, characterizing the efficient allocation in a general environment is intractable. So we resort to a class of examples to illustrate the comparative advantage of bank equity capital. The probability that any given technology receives a negative productivity shock is relatively small but the probability that one or more technologies receives a shock is relatively large. A diversified bank has a comparative advantage in providing a buffer against these shocks. First, it only needs to hold a small amount of equity because, in a typical state, only a small fraction of its portfolio is affected. Second, because there is often some type of firm that is defaulting, the bank's equity buffer is more likely to be required than a firms' equity buffer. Both banks and firms will normally find it privately and socially optimal to issue equity, of course.

The rest of the paper is organized as follows. In Section 2 we present the economy and the competitive equilibrium notion. In Section 3 we show the welfare properties of equilibria. Section **??** examines the properties of banks' capital structure in equilibrium. First, Section **??** shows that bank capital is positive in the single factor model with idiosyncratic risk and diversified portfolios. Section 4.2 shows that when the technologies are co-monotonic, bank equity has zero value in equilibrium. Then Section 4.3 characterizes the equilibrium capital structures in an environment where productivity shocks have an idiosyncratic component, finding that they feature a positive value of equity. Section 5 concludes. Proofs are collected in the appendix.

## 1.1 Related literature

Modigliani and Miller (1958) provides a benchmark in which capital structure is indeterminate and has no effect on the value of the firm. A large literature has grown up investigating the role of various factors, such as taxes, bankruptcy, term structure, seniority and incentive problems, in the choice of corporate capital structure (Brennan and Schwartz, 1978; Barnea, Haugen and Senbet, 1981; Kim, 1982; Titman, 1984; Dammon and Green, 1987; Titman and Wessels, 1988; Leland and Toft, 1996; Bradley, Jarrell, and Kim, 2011; Hackbarth and Mauer, 2012). In our model, the optimal capital structure is determined by a tradeoff off between the default costs of debt and the funding advantage of debt, rather than the tax hedge of interest on debt.

The empirical literature shows that costs of default can be substantial for both banks and non-financial firms (James, 1991; Andrade and Kaplan, 1998; Korteweg, 2010). More recent work suggests that earlier estimates may have understated the true costs of default (Almeida and Philippon, 2007; Acharya, Bharath and Srinivasan, 2007). The default costs in the present are are a deadweight loss, as distinct from the fire sale "losses" in Gale and Gottardi (2015), which are merely transfers of value to buyers.

Because of default costs and the liquidity premium on deposits, the Modigliani Miller Theorem does not hold in our model. The empirical literature on the relationship between a bank's capital structure and its market value is not large. Flannery and Rangan (2008) examined changes in banks' capital structure in the previous decade. Mehran and Thakor (2011) found a positive relationship between bank value and bank capital in a cross section of banks. Gropp and Heider (2010) found that the determinants of bank capital structure were similar to those of non-financial firms, although the levels of equity are different.

In the theoretical literature on banking, Diamond and Dybvig (1983) and Diamond (1984) show that deposits constitute the optimal form of funding to provide liquidity insurance to depositors or delegated monitoring for investors. Gale (2004) extends the Diamond-Dybvig model to include bank capital that provides additional risk sharing between risk neutral equity holders and risk averse depositors. The importance of the liquidity services provided by deposits has been emphasized, more recently, by Stein (2012) and De Angelo and Stulz (2015).

Van den Heuvel (2008) studies a quantitative model in which bank capital structure is determined by the trade-off between the liquidity services of bank debt and the costs of moral hazard that are associated with risk shifting behavior. The model does not allow for aggregate uncertainty and assumes that deposits yield direct utility benefits. DeAngelo and Stulz (2015) also highlight the liquidity premium earned by bank deposits, contrasting it with the costs of intermediation.

Gornall and Strebulaev (2015) provide a quantitative analysis of a model in which the capital structures of banks and borrowers are endogenously determined. They show that the optimal leverage in the banking sector is much higher than in the corporate sector. The banks are less risky than the borrowers for two reasons. First, the banks hold senior debt claims, so the first loss falls on the corporate shareholders. Second, the banks reduce the risk of their portfolio by diversifying across firms. These two factors are sufficient to produce realistic levels of bank capital.

Allen, Carletti and Marquez (2015), henceforth ACM, motivated the present paper. ACM construct a model in which the Modigliani Miller Theorem does not hold and characterize the equilibrium capital structures of firms and banks. Like us, ACM assume banks and firms can only issue debt (deposits) and equity. They also assume that markets for deposits and equity are segmented: some consumers can only hold deposits, whereas others can hold equity. Depositors have lower outside options than equity investors, so in equilibrium the depositors receive a lower return than the equity investors. Equity is therefore an "expensive" source of funding.

ACM assume the capital structure of the representative firm and bank are chosen jointly to maximize total expected surplus, taking as given the expected returns demanded by the holders of equity and deposits, respectively. This optimal contracting approach guarantees the efficiency of equilibrium capital structures. In our framework, by contrast, efficiency is a property of a decentralized competitive equilibrium when markets are complete.

ACM also derive the result that bank equity has zero value, for the special case where firms' returns are perfectly correlated and uniformly distributed. In that case, the bank is simply a pass-through for the shocks affecting the firms' returns and the bank will default only if the firms default. As a result, putting all the equity in the firms minimizes the probability of default for both banks and firms.

We should also mention a large theoretical literature on the role of bank capital in preventing risk shifting or asset substitution, beginning with the seminal paper of Stiglitz and Weiss (1981) and including recent contributions such as Martinez-Miera and Repullo (2010).

Our competitive equilibrium model is related to the literature on the theory of the firm in incomplete markets, developed by Diamond (1967), Ekern and Wilson (1974), Radner (1974), Drèze (1974), and Grossman and Hart (1979). In the earlier literature, firms are fully owned by shareholders and the equilibrium value of a firm is determined by the marginal valuations of its owners. For example, a firm that produces a vector of future outputs $\mathbf{y}$ has market value

$$v = \frac{\nabla u_i(\mathbf{x}_i) \cdot \mathbf{y}}{\|\nabla u_i(\mathbf{x}_i)\|}$$

where $\mathbf{x}_i$ is the shareholder's consumption bundle and $\nabla u_i(\mathbf{x}_i)$ is the vector of marginal utilities. Our assumption of complete markets for debt and equity implies the existence of equilibrium prices for all possible securities, even those that are not traded in equilibrium. A similar approach is found in Makowski (1983) and Allen and Gale (1988, 1991). An alternative to the complete markets approach is to assume that only traded securities are priced, but that firms have rational conjectures about the value a security would have if a small amount of it were introduced. This approach was used by Hart (1979), for example, and appears to give the same results as the complete markets approach under sufficiently strong regularity conditions.

The existence of intermediaries and the costs of default make the pricing of assets more complicated in our model than in a stock market economy. Because a firm's debt is held by banks and default can occur at the firm level, the bank level, or both, the value of a firm's debt will depend on banks' willingness to pay for it, which in turn depends on the banks' capital structure and the consumers' willingness to pay for the debt and equity of banks. Bisin, Gottardi and Ruta (2014) also study the pricing of securities when intermediaries are present.

# 2 A general equilibrium model of capital structure

## 2.1 Endowments and technologies

There are two dates, indexed $t = 0, 1$, and a finite number of states of nature, $s = 1, ..., S$. The true state is unknown at date 0 and revealed at date 1. The probability of state $s$ at date 0 is denoted by $\pi_s > 0$, for $s = 1, ..., S$.

There are two goods, a non-produced capital good and a produced consumption good. Consumption is produced subject to constant returns to scale using capital goods as the only input. There is a finite number of technologies, indexed $j = 1, ..., n$, for producing the consumption good. Using technology $j$, one unit of capital at date 0 produces $A_{js} > 0$ units of consumption at date 1 in state $s$.

There is a continuum of identical consumers with unit mass. Each consumer has an initial endowment of $k_0 = 1$ units of capital at date 0. There is no initial endowment of consumption.

## 2.2 Firms

We assume that each active firm can invest in only one of the $n$ technologies available. Because production is subject to constant returns to scale, we can focus without loss of generality on the case where each firm uses one unit of capital. The amount of capital invested in each technology is then equal to the number of firms using that technology.

In this environment, a firm's capital structure is determined by the face value of the debt it issues. The face value of the debt is denoted by $\ell$ and is assumed to belong to a finite interval $L = [0, \ell_{\max}]$.

Because productivity shocks are the only source of uncertainty, the technology choices made by firms determine the level of risk in the economy, while their capital structure choices determine how this risk is distributed between debt and equity.

Firms are identical ex ante but they may differ in their choice of technology and capital structure. A firm's choice of face value of debt $\ell$ and technology $j$ is referred to as the firm's *type*. The set of firm types is denoted by $F \equiv L \times N$, with generic element $(\ell, j)$, where $N = \{1, ..., n\}$ denotes the set of available technologies. Although the number of possible types is infinite, we focus on equilibria in which the number of active types is finite.[6]

A firm of type $f = (\ell, j) \in F$ issues debt and equity. The payoff vectors of these assets, denoted, respectively, by $\mathbf{a}_f^d \in \mathbf{R}_+^S$ and $\mathbf{a}_f^e \in \mathbf{R}_+^S$, are uniquely determined by the firm's type $f = (\ell, j)$ as follows:

$$a_{fs}^d = \begin{cases} \ell & \text{if } A_{js} \geq \ell \\ \lambda_f A_{js} & \text{if } A_{js} < \ell \end{cases} \tag{1}$$

and

$$a_{fs}^e = \begin{cases} A_{js} - \ell_j & \text{if } A_{js} \geq \ell \\ 0 & \text{if } A_{js} < \ell \end{cases} \tag{2}$$

for any state $s$. The parameter $0 \leq \lambda_f < 1$ is the firm's *recovery ratio* in the event of default. In other words, the default costs are $1 - \lambda_f$ per unit of output. For generality, we allow the recovery ratio to depend on the firm's type $f$, but this is not necessary and, in most applications, $\lambda_f$ is independent of $f$.

Firms choose their technology and capital structure to maximize their profits, which is equivalent to maximizing the firm's market value. Since firms are subject to constant returns to scale, profits must be zero in equilibrium. In other words, the market value of the securities issued by a firm is just enough to finance the purchase of capital goods. Types of firms that cannot earn a zero profit will not operate in equilibrium.

Securities issued by firms are sold on competitive markets. In line with our completeness assumption, there is a price at which the securities issued by each type of firm are traded. Prices are denoted by the vector $\mathbf{q}_F = \left(\mathbf{q}_F^d, \mathbf{q}_F^e\right) \in \mathbf{R}_+^F \times \mathbf{R}_+^F$, where $\mathbf{q}_F^d$ is the subvector of debt prices and $\mathbf{q}_F^e$ is the subvector of equity prices. The *market value* of a firm of type $f$ then is $q_f^d + q_f^e$. We normalize the price of capital goods to be equal to one. Hence, in equilibrium, we have $q_f^d + q_f^e \leq 1$ for any $f \in F$—otherwise the demand for capital goods would be unbounded—and only the firm-types that achieve zero profits, $q_f^d + q_f^e = 1$, will operate in equilibrium.

[6] The number of active types needed for existence is finite by Caratheodory's Theorem.

## 2.3 Banks

Banks lend to firms by purchasing their debt. We assume banks do not invest in firm equity.[7] Banks raise funds by issuing deposits and equity to consumers. A bank's capital structure is determined by the level of deposits it chooses to issue. We denote the face value of deposits by $d \in D$, where $D$ is a finite interval $[0, d_{\max}]$. The bank's portfolio is described by a vector $\mathbf{x} \in \mathbf{R}_+^F$, where $x_f \geq 0$ denotes the units of debt of type-$f$ firms held by the bank.[8] Since the banks' technology is subject to constant returns to scale, we can assume without loss of generality that each bank's portfolio is normalized so that $\sum_{f \in F} x_f = 1$. In other words, the bank invests in a portfolio of debt issued by firms that collectively operate one unit of capital goods. Let $X \subset \mathbf{R}_+^F$ denote the set of admissible debt portfolios.

All banks have access to all types of firms' debt and the same funding opportunities. Their portfolios and capital structures may differ, however. We refer to a bank's portfolio $\mathbf{x}$ and capital structure $d$ as its *type*. The set of bank types is $B = X \times D$. Again, only a finite number of bank types will be active in equilibrium. Let $\mathbf{x}_b$ denote the portfolio of a bank of type $b$. The payoff vectors of the deposits and equity issued by a bank of type $b$ are denoted by $\mathbf{a}_b^d \in \mathbf{R}_+^S$ and $\mathbf{a}_b^e \in \mathbf{R}_+^S$, respectively, and defined by

$$a_{bs}^d = \begin{cases} d & \text{if } \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d \geq d \\ \lambda_b \left( \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d \right) & \text{if } \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d < d \end{cases} \tag{3}$$

and

$$a_{bs}^e = \begin{cases} \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d - d & \text{if } \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d \geq d \\ 0 & \text{if } \mathbf{x}_b \cdot \mathbf{a}_{Fs}^d < d, \end{cases} \tag{4}$$

for every state $s$, where the vector $\mathbf{a}_{Fs}^d$ is defined by

$$\mathbf{a}_{Fs}^d = \left( \mathbf{a}_{fs}^d \right)_{f \in F}$$

for each $s$. Note that the payoff vectors are completely determined by the bank's type, as in equations (3) and (4). The recovery rate $0 \leq \lambda_b \leq 1$ is a constant and may or not depend on the bank's type $b \in B$.

The problem of each bank is to select its portfolio and its capital structure to maximize its profits, given by the difference between its market value, that is the value of the liabilities it issued, and the value of the portfolio it acquired. The bank takes as given the price of all debt claims issued by firms, $\mathbf{q}_F^d \in \mathbf{R}_+^F$, as well as the prices of all types of securities the

[7] This ensures the presence of a nontrivial interdependence between the capital structure of firms and banks. Also, current regulations make this assumption realistic.

[8] In equilibrium, each bank will lend to a finite number of types of firms, since the number of active firm types is finite.

banks can issue, $\mathbf{q}_B = \left(\mathbf{q}_B^d, \mathbf{q}_B^e\right) \in \mathbf{R}_+^B \times \mathbf{R}_+^B$, where $\mathbf{q}_B^d$ is the subvector of deposit prices and $\mathbf{q}_B^e$ is the subvector of equity prices. More formally, each bank will choose its type $b \in B$ to maximize market value minus the cost of the assets it acquired, $q_b^d + q_b^e - \mathbf{q}_F^d \cdot \mathbf{x}_b$. In equilibrium, the maximum profit will be zero, that is, $q_b^d + q_b^e \leq \mathbf{q}_F^d \cdot \mathbf{x}_b$, and only banks that achieve zero profits, $q_b^d + q_b^e = \mathbf{q}_F^d \cdot \mathbf{x}$, will be active.

## 2.4 Consumers

All consumption takes place at date 1, when the output of consumption good is realized. Consumers have VNM preferences over consumption described by

$$\sum_s \pi_s u \left(c_{1s} + \beta c_{2s}\right), \tag{5}$$

where $c_{1s}$ denotes the consumption in state $s$ that can occur immediately, because it is paid for with deposits, while $c_{2s}$ denotes the consumption which is paid for with the yields of equity, and occurs with some delay. The constant $0 < \beta < 1$ captures the cost of this delay. The specification of the preferences reflects the assumption that deposits serve as money, whereas equity does not. The delay (or equivalently, transaction) costs involved in converting equity into "cash" are measured by the parameter $\beta \in (0, 1)$.[9] The function $u : \mathbf{R}_+ \to \mathbf{R}$, describing the utility of total consumption in any state $s$, $c_{1s} + \beta c_{2s}$, is assumed to be increasing, concave and continuously differentiable.

Each consumer can use the revenue obtained by selling his endowment of capital at date 0 to purchase the deposits and equity issued by banks and the equity issued by firms. Consumers cannot purchase firm debt directly, but hold it indirectly by investing in banks that purchase firm debt.[10]

A consumer's portfolio is described by a vector $\mathbf{z} \equiv (\mathbf{z}_F, \mathbf{z}_B) \in \mathbf{R}_+^F \times \mathbf{R}_+^F \times \mathbf{R}_+^B \times \mathbf{R}_+^B$, where $\left(z_b^d, z_b^e\right)$ denotes the consumer's demand for debt and equity issued by banks of type $b$ and similarly for $\left(z_f^d, z_f^e\right)$. Although consumers have access to an infinite number of securities, the consumer's portfolio must have a finite support in equilibrium. The set of feasible portfolios is denoted by $Z$ and defined to be the set of portfolios $\mathbf{z}$ with finite support such that $z_f^d = 0$ for all $f \in F$. Letting $\mathbf{q} \equiv (\mathbf{q}_F, \mathbf{q}_B)$, the consumer chooses a consumption bundle

[9] The specification is a reduced-form representation of the greater convenience of using deposits for consumption compared to equity. A shareholder who wants to convert shares into consumption must pay a commission to sell the shares. Dividends are paid infrequently and must be converted into deposits before they can be spent. This time delay reduces the value of the consumption because of discounting.

[10] Although we maintain the assumption for simplicity, it seems quite realistic. Banks may have an advantage in monitoring firms and enforcing repayment of loans. And since loans to firms do not function as "money," deposits are more attractive to consumers in any case.

$\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2) \in \mathbf{R}^S_+ \times \mathbf{R}^S_+$ and a portfolio $\mathbf{z} \in Z$ to maximize

$$U(\mathbf{c}) \equiv \sum_{s=1}^{S} \pi_s u(c_{1s} + \beta c_{2s})$$

subject to the budget constraints,

$$\begin{aligned} \mathbf{q} \cdot \mathbf{z} &\leq 1, \\ \mathbf{c}_1 &= \sum_{b \in B} z^d_b \mathbf{a}^d_b, \\ \mathbf{c}_2 &= \sum_{b \in B} z^e_b \mathbf{a}^e_h + \sum_{f \in F} z^e_f \mathbf{a}^e_h. \end{aligned}$$

## 2.5 Equilibrium

An allocation is described by a consumption bundle, $\mathbf{c}$, and a portfolio, $\mathbf{z}$, of the representative consumer, a distribution of banks over the set of possible bank types $\boldsymbol{\mu} = (\mu_b)_{b \in B}$, and a distribution of firm types $\boldsymbol{\kappa} = (\kappa_f)_{f \in F}$. Formally, the *allocation* is an array $(\mathbf{c}, \mathbf{z}, \boldsymbol{\mu}, \boldsymbol{\kappa})$, where $\mathbf{z}$, $\boldsymbol{\mu}$, and $\boldsymbol{\kappa}$ have finite supports. An allocation is *attainable* if

$$\sum_{f \in F} \kappa_f = 1, \tag{6}$$

$$\sum_{b \in B} \mu_b \mathbf{x}_b = \boldsymbol{\kappa}, \tag{7}$$

$$z^d_b = z^e_b = \mu_b, \ \forall b \in B, \tag{8}$$

$$z^e_f = \kappa_f, \ \forall f \in F, \tag{9}$$

and

$$\mathbf{c} = \mathbf{z} \cdot \mathbf{a} = \left( \sum_{b \in B} z^d_b \mathbf{a}^d_b, \sum_{b \in B} z^e_b \mathbf{a}^e_b + \sum_{f \in F} z^e_f \mathbf{a}^e_f \right). \tag{10}$$

The first attainability condition (6) says that the firms collectively use the entire one unit of the capital good in the consumers' endowments. The second condition, (7), says that banks hold in their portfolio all the debt issued by firms. The third and fourth conditions, (8) and (9), say that consumers hold all the deposits and equity issued by banks and all the equity issued by firms. Finally, the last condition, (10), restates the relationship between consumption and the payoff of the portfolio held by consumers.

An *equilibrium* consists of an attainable allocation $(\mathbf{c}, \mathbf{z}, \boldsymbol{\mu}, \boldsymbol{\kappa})$ and a price system $\mathbf{q}$ such that:

(i) $\kappa_f > 0$ only if $f$ solves the firm's problem, given the prices $\mathbf{q}$;

(ii) $\mu_b > 0$ only if $b$ solves the bank's problem, given the prices $\mathbf{q}$;

(iii) $(\mathbf{c}, \mathbf{z})$ solves the consumer's problem, given the prices $\mathbf{q}$.

Note that equilibrium condition (i) is equivalent to

$$\kappa_f > 0 \implies q_f^d + q_f^e = \max_{f \in F} \left\{ q_f^d + q_f^e \right\} = 1,$$

for any $f \in F$. Similarly, equilibrium condition (ii) is equivalent to

$$\mu_b > 0 \implies q_b^d + q_b^e - \mathbf{q}_F^d \cdot \mathbf{x} = \max_{b \in B} \left\{ q_b^d + q_b^e - \mathbf{q}_F^d \cdot \mathbf{x} \right\} = 0,$$

for any $b \in B$. In what follows, we refer to a firm of type $f$ (respectively, bank of type $b$) as being *active* in equilibrium if and only if $\kappa_f^* > 0$ (respectively, $\mu_b^* > 0$). Also, prices are such that markets for the securities of non active firms clear with zero trades.

# 3 Constrained efficiency

In this section, we show that analogues of the Fundamental Theorems of Welfare Economics hold for the environment described above. Markets are incomplete, since banks and firms are restricted to using debt and equity, so the appropriate welfare concept is *constrained* Pareto efficiency, rather than Pareto efficiency.

We say that an attainable allocation $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ is *constrained Pareto efficient*, or constrained efficient, for short, if there does not exist an attainable allocation $(\mathbf{c}, \mathbf{z}, \boldsymbol{\mu}, \boldsymbol{\kappa})$ such that $U(\mathbf{c}) > U(\mathbf{c}^*)$. Formally, this is the case if and only if $(\mathbf{c}^*, \boldsymbol{\mu}^*)$ solves the problem

$$\max_{(\mathbf{c}, \boldsymbol{\mu})} \sum_{s=1}^{S} \pi_s u(c_{1s} + \beta c_{2s})$$

subject to the constraints

$$\sum_{b \in B} \mu_b = 1, \tag{11a}$$

$$\mathbf{c} = \sum_{b \in B} \mu_b \left( \mathbf{a}_b^d, \mathbf{a}_b^e \right) + \sum_{b \in B} \mu_b \mathbf{x}_b \cdot \mathbf{a}_F^e. \tag{12a}$$

To see this, note first that if $(\mathbf{c}^*, \boldsymbol{\mu}^*)$ satisfies the constraints (11a) and (12a), we can use the attainability conditions (8) and (9) to define the consumers' portfolio $\mathbf{z}$ and use the attainability condition (7) to define $\kappa_f$. Then it is easy to check that $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ satisfies the attainability constraints (6)–(10). Conversely, if $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ is an attainable allocation, $(\mathbf{c}^*, \boldsymbol{\mu}^*)$ satisfies the constraints.[11]

**Proposition 1** *Let* $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*, \mathbf{q}^*)$ *be an equilibrium. Then* $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ *is constrained Pareto efficient.*

The argument of the proof is standard, and exploits the fact that markets for all the possible types of securities that can be issued by firms and banks are competitive and complete. Also, note that the set of attainable consumption vectors satisfying (11a), (12a) is convex and this allows us to establish the following:

**Proposition 2** *Suppose that* $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ *is a constrained efficient allocation. Then there exists a price vector* $\mathbf{q}^*$ *such that* $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*, \mathbf{q}^*)$ *is an equilibrium.*

Although these results look quite "standard," they require a number of restrictive assumptions. Without a complete set of markets for contingent claims, we can only ensure the equilibrium is constrained efficient. As Geanakoplos and Polemarchakis (1986) have shown, a competitive equilibrium with incomplete markets is generically constrained inefficient unless special conditions are satisfied. One of these conditions is the existence of a single representative consumer; another is the assumption of a single good. These assumptions are common in financial applications, but they are nonetheless restrictive. The representative consumer assumption is not crucial—as long as there is a single good, we could extend the theory to allow for multiple types of consumers—but the single good assumption would be harder to remove. Finally, the assumption that markets are open for all securities is crucial for the coordination of capital structure choices in equilibrium.[12] The resulting benchmark model is important for two reasons: first, it demonstrates the possibility of efficient coordination of capital structures in a decentralized economy and, second, it considerably simplifies the characterization of the equilibrium capital structures, because they are the solutions of a planner's problem.

[11] To show this, we simply need to use the attainability conditions (7), (8), and (9) to eliminate $\mathbf{z}^*$ and $\boldsymbol{\kappa}^*$ from (10), getting constraint (12a) as a result. Similarly, the attainability conditions (6) and (7) imply constraint (11a).

[12] Hart (1979) follows an alternative approach, in which the equilibrium of a stock market economy is reached in two stages. In the first stage, firms choose production plans and have rational expectations about the value of the firm that will be realized in the second stage. This approach only requires markets to open for the shares of firms that actually form. Under sufficient regularity conditions, this appears to be equivalent to the approach adopted here.

# 4 Equilibrium capital structures

We have shown that the equilibrium capital structures are constrained efficient, but have not attempted to characterize the equilibrium capital structures. This is a very difficult problem because of the non-convexities introduced by the use of debt and the deadweight costs of default. In this section we explore the interaction between the choice of capital structures by firms and banks. Both banks and firms have incentives to include equity in their capital structures because equity provides a buffer against default. But a firm's equity does "double duty" because it makes the firm's debt less risky and that in turn reduces the riskiness of the bank's assets. If a firm has very low leverage, its debt will be relatively safe. If a bank lends only to firms with low leverage, the bank's assets will be relatively safe and the need for an equity buffer will be diminished. This suggests that it may be efficient for banks to have higher leverage (lower equity buffers) than firms. Gornall and Strebulaev (2015) use a quantitative model to calculate the optimal (value maximizing) capital structures of firms and banks and find that the representative bank will have higher leverage than the representative firm. They attribute the relatively high leverage of banks to two facts: first, banks hold a senior claim (debt) on the firms' cash flows and, second, they hold diversified portfolios. But it is also clear that firm equity and bank equity are substitutes and, for some purposes, firm equity may be more effective.

As we noted earlier, ACM develop a general equilibrium model and show that, for some parameter values, the efficient capital structure entails no bank equity at all. The ACM result is special, of course, but emphasizes the substitutability of bank equity and firm equity. In the remainder of this section, we investigate the comparative advantages of bank equity and firm equity and the extent to which they are complements. We begin in the next subsection by considering a single factor model and showing conditions under which bank equity must be positive.

## 4.1 A "single factor" model

In the banking literature, single factor models, such as the one due to Vasicek (2002), are widely used to represent loss distributions of bank portfolios. Single factor models have also been incorporated in equilibrium models, such as Martinez-Miera and Repullo (2010), in order to study the effect of capital requirements on bank failure. In single factor models, the value of the assets $A_j(T)$ of a borrowing firm $j$ at time $T$ are assumed to be the product of two random variables,

$$\log A_j(T) = \log A_j(0) + \log \sqrt{\rho} z + \log \sqrt{1-\rho} \varepsilon_j,$$

where $z$ is a standard normal aggregate shock, $\varepsilon_j$ is a standard normal idiosyncratic shock, $\rho$ is the correlation parameter, and $z$ and $\{\varepsilon_j\}$ are i.i.d. random variables whose means and variances are functions of $T$. Firm $j$ defaults at time $T$ if $B_j > A_j(T)$, where $B_j$ is the face value of the loan given by the bank.

We can obtain something similar in our model, if we assume that there is a large number of technologies $j \in [0,1]$. The productivity of technology $j$ (per unit of capital) is given by $A_j = \theta_j a$, where $\theta_j$ and $a$ are random variables. Let $\theta$ and $a$ be distributed according to $F(\theta)$ and $G(a)$ respectively, where $F$ and $G$ are $C^1$, both have support $[0,\infty)$, $f(\theta) > 0$ for $\theta > 0$ and $g(a) > 0$ for $a > 0$, where $f(\theta)$ and $g(a)$ are the respective probability density functions. For the purpose of this exercise, we assume that all firms have the same face value of the debt $\ell \geq 0$ and the representative bank holds a perfectly diversified portfolio across the different technologies. Let $\ell(a)$ be the amount repaid when the aggregate shock is $a$. Then

$$\ell(a) = \int_0^{\ell/a} \lambda_f \theta a f(\theta)\, d\theta + (1 - F(\ell/a))\,\ell,$$

for any value of $a$. Under the maintained assumptions, $\ell(a)$ is increasing in $a$. In fact,

$$\begin{aligned} \ell'(a) &= (\lambda_f \ell f(\ell/a) - \ell f(\ell/a)) \frac{-\ell}{a^2} \\ &= (1-\lambda_f)\left(\frac{\ell}{a}\right)^2 f(\ell/a) > 0. \end{aligned}$$

Let $d(a)$ denote the value of withdrawals if the aggregate shock is $a$. Then

$$d(a) = \begin{cases} d & \text{if } d < \ell(a) \\ \lambda_b \ell(a) & \text{if } d > \ell(a). \end{cases}$$

and the return to bank equity is $\max\{\ell(a) - d, 0\}$. The return to firm equity, if the aggregate shock is $a$, is

$$\int_{\ell/a}^{\infty} (\theta a - \ell) f(\theta)\, d\theta.$$

Assume the representative agent is risk neutral. Then an efficient choice of $\ell$ and $d$ must maximize

$$\int_0^{\infty} \left\{ d(a) + \beta \max\{\ell(a) - d, 0\} + \beta \int_{\ell/a}^{\infty} (\theta a - \ell) f(\theta)\, d\theta \right\} g(a)\, da. \tag{13}$$

By the usual argument, we must have $d \leq \ell$ in equilibrium and the expected returns to bank equity will be positive if and only if $d < \ell(a)$ with positive probability. But this must always be true if $d < \ell$. To see this note that $\ell(a) < \ell$ for any finite value of $a$ if $\ell > 0$ and $\ell(a) \to \ell$ as $a \to \infty$. So if $d < \ell$, there is positive probability that $d < \ell(a)$ and the value of the bank's equity is positive.

**Proposition 3** *Under the maintained assumption, the values of $d$ and $\ell$ that maximize (13) must satisfy $d < \ell$.*

Why does the efficient bank capital structure have a positive value of equity in the single factor model? It has to do with the idiosyncratic shock $\theta$. If $\theta$ is a degenerate random variable, for example, $\theta = 1$ with probability one, the equity buffer in the firms' capital structure would be sufficient and the constrained efficient value of bank equity would be zero. This result holds for the single factor model but it also holds for the general equilibrium model with multiple technologies.

## 4.2 Co-monotonic technologies

In this section, we assume the $n$ available technologies are *co-monotonic*.

**Definition 4** *Technologies are said to be co-monotonic if $A_{js-1} < A_{js}$, for every $s = 2, ..., S$ and $j = 1, ..., n$.*

This condition requires that the productivities of all technologies are increasing functions of the state $s$. In other words, the productivity shocks are driven by a single factor and there is no idiosyncratic component. As a consequence, an increase in $s$ reduces defaults for all types of firms and banks. With this assumption, we obtain a generalization of the ACM result.[13]

**Proposition 5** *Assume that technologies are co-monotonic. Then if $(c^*, z^*, \mu^*, \kappa^*, q^*)$ is an equilibrium, the value of bank equity is zero for all* active bank types *$b \in B$.*

The co-monotonicity assumption is stated as a property of the productivity of all the technologies available in the economy. It is easy to see from the proof of Proposition 5 that this result is valid as long as the bank lends only to firms with co-monotonic technologies. For any bank portfolio $\mathbf{x}$, let the set of technologies represented in the portfolio be denoted by $J(\mathbf{x})$ and defined by

$$J(\mathbf{x}) = \left\{ j = 1, ..., n : x_{(\ell,j)} > 0 \text{ for some } \ell \right\}.$$

Then we say that the portfolio $\mathbf{x}$ is *co-monotonic* if the set of technologies $J(\mathbf{x})$ is co-monotonic in the usual sense. The following corollary is then immediate.

[13] ACM make a number of other restrictive assumptions not required in our framework: consumers are risk neutral and exogenously divided into depositors and shareholders; there is a single technology with uniformly distributed productivity; and banks and firms choose their capital structures to maximize total surplus.

**Corollary 6** *In any equilibrium* $(c^*, z^*, \mu^*, \kappa^*, q^*)$*, the value of equity is zero for any active bank* $b^*$ *whose portfolio* $\mathbf{x}^*$ *is co-monotonic.*

So now we know that idiosyncratic risk is necessary for bank equity to positive value, but is it sufficient outside the assumptions of the single factor model? In the next section, we study a simple example that allows us to see more clearly some of the factors that play a role in determining the efficient capital structure.

## 4.3 Positive bank equity

In this section we explore environments with both aggregate and idiosyncratic productivity shocks and identify conditions under which constrained efficiency requires a positive value of bank equity. We will assume that consumers are risk neutral, which simplifies the characterization of equilibrium prices and quantities. Under this assumption, an attainable allocation $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ is constrained efficient if and only if any type of bank $b^*$ in the support of $\boldsymbol{\mu}^*$ satisfies

$$\sum_{s=1}^{S} \pi_s \left(c_{1s}^* + \beta c_{2s}^*\right) = \sum_{s=1}^{S} \pi_s \left(a_{b^*s}^d + \beta \left(a_{b^*s}^e + \mathbf{x}_{b^*}^* \cdot \mathbf{a}_F^e\right)\right).$$

In other words, if we think of an active bank and the firms that borrow from it as a conglomerate, the market value of this conglomerate must equal the expected value of consumption for the representative consumer. If this condition were not satisfied, either the bank or the firms or both would not be maximizing their market values. We use this property repeatedly in what follows.

We consider a simple environment in which it is possible to derive the equilibrium capital structures explicitly. There are $n$ technologies and a finite number $S = n + 2$ of states of nature. The probability of state $s$ is denoted by $\pi_s$ and given by

$$\pi_s = \begin{cases} \frac{1-\delta-\varepsilon}{n} & \text{for } 1 \leq s \leq n \\ \delta & \text{for } s = n+1 \\ \varepsilon & \text{for } s = n+2 \end{cases}. \tag{14}$$

The productivity of technology $j$ in state $s$ is assumed to satisfy

$$A_{js} = \begin{cases} a_L & \text{if } s = j \leq n \\ a_M & \text{if } s \neq j \leq n \\ a_M & \text{if } s = n+1 \\ a_H & \text{if } s = n+2 \end{cases}, \tag{15}$$

where $0 < a_L < a_M < a_H$.

To compare this to the single factor model, we can represent the productivity of technology by $A_{js} = \theta_j A$, where $A$ is an aggregate shock and $\theta_j$ is an idiosyncratic shock. The marginal distributions are given by

$$A = \begin{cases} a_H & \text{w. prob. } \delta \\ a_M & \text{w. prob. } 1-\delta \end{cases}$$

and

$$\theta_j = \begin{cases} 1 & \text{w. prob. } \delta \\ 1 & \text{w. prob. } 1-\delta-(n-1)\,\varepsilon/n \\ a_L/a_M & \text{w. prob. } \varepsilon/n \end{cases} .$$

In the single factor model of Section **??**, we assumed that the shock $\theta_j$ was i.i.d. across technologies and independent of $A$. Here we assume that the idiosyncratic shock $\theta_j$ is degenerate and equal to one when $A = a_H$ takes the values 1 and $a_L/a_M$ when $A = a_M$. In addition, we assume the idiosyncratic is negatively correlated across technologies, in the sense that it is equal to $a_L/a_M < 1$ for at most one technology $j$ in any state. These assumptions are made to simplify the analysis, which is quite lengthy even with these simplifying assumptions.

A bank portfolio is said to be *simple* if all firms with the same technology, whose debt is held by the bank, have the same capital structure. We can show that it is optimal for banks to choose a simple portfolio when the structure of technologies satisfies (14) and (15). Since the technologies are ex ante identical, it follows that the same capital structure is optimal for all firms.

**Proposition 7** *Suppose the technologies satisfy (14) and (15). Then, in equilibrium, every bank chooses a simple portfolio, that is, one containing either (i) only the debt of firms with no default risk ($\ell_j = a_L$), or (ii) only the debt of firms with low default risk ($\ell_j = a_M$), or (iii) only the debt of firms with high default risk ($\ell_j = a_H$).*

Given the technology structure, there are three possible candidates for the face value of the firm's debt. The firm can choose the face value equal to $a_L$, so that it never defaults, or $a_M$, so that the firm only defaults when hit by a negative shock, or $a_H$, so that the firm defaults unless it is hit by a large positive shock. Proposition 7 assures us that a bank will lend to firms that use only one of these capital structures. Unfortunately, there is no intuitive explanation for this result. The proof proceeds by considering all possible portfolios and showing that non-simple portfolios are always dominated.

The specification given by (14) and (15) incorporates a number of interesting cases. In the limit as $\delta + \varepsilon \to 1$, all technologies are identical (there is only aggregate risk) and, hence, co-monotonic. Then Proposition 5 implies that the value of bank equity will be zero in any equilibrium when $\delta + \varepsilon = 1$. At the other extreme, in the limit as $\delta \to 0$ and $\varepsilon \to 0$, we have the case of pure idiosyncratic risk. In each of the baseline states $\{1, .., n\}$, exactly one technology yields $a_L$ and the remainder yield $a_M$. In this case too, we show that bank equity has zero value, but for rather different reasons than in the co-monotonic case. The following proposition characterizes the equilibrium allocations in the idiosyncratic case. These allocations are, of course, constrained efficient by Proposition 1.

**Proposition 8** *Assume the technologies satisfy (14), (15) and $\delta = \varepsilon = 0$. Then in equilibrium banks' choices are as follows:*
*(i) if $\left(\frac{n-\lambda_f}{n-1}\right) a_L > a_M$, each bank lends exclusively to firms with safe debt ($\ell_j = a_L$), and sets its level of deposits at $d = a_L$,*[14] *or*
*(ii) if $\left(\frac{n-\lambda_f}{n-1}\right) a_L < a_M$, each bank lends exclusively to firms with risky debt ($\ell_j = a_M$) and chooses a fully diversified portfolio (holding $1/n$ units of the debt of firms choosing technology $j$, for each $j \in \{1, .., n\}$) and sets*

$$d = \frac{n-1}{n} a_M + \frac{1}{n} \lambda_f a_L.$$

*In either case bank equity has zero value.*

When $\delta = \varepsilon = 0$, there are two possible equilibrium allocations, distinguished by the face value of firms' debt and by whether or not firms default. In each equilbrium allocation, banks' portfolios are risk free. Note that a risk free portfolio should always be entirely funded with deposits because equity is costly ($\beta < 1$) and there is no need for an equity buffer in absence of default risk.

In both of the extreme cases in our example, that is, when $\delta = \varepsilon = 0$ and $\delta + \varepsilon = 1$, it is optimal to have zero bank equity. In between the two extremes, when there is both aggregate risk and idiosyncratic risk, there is a role for equity in the efficient capital structures of both firms and banks. The following result states this more precisely.

**Proposition 9** *Assume the technologies satisfy (14) and (15). Then if*

$$\left(\frac{n-\lambda_f}{n-1}\right) a_L < a_M,$$

*there exist $\delta^* > 0$ and $\varepsilon^* > 0$ such that, for any $0 < \delta < \delta^*$ and $0 \leq \varepsilon < \varepsilon^*$, in any equilibrium, each bank chooses a fully diversified portfolio, investing in loans with face value $\ell = a_M$, to firms with $1/n$ units of capital, for each technology $j = 1, ..., n$, and the face value of deposits is*

$$d = \frac{n-1}{n} a_M + \frac{1}{n} \lambda_f a_L.$$

[14] The choice technology is immaterial since safe debt pays $a_L$ with probability one.

Obviously, bank equity has positive value because a bank receives a loan repayment equal to $a_M > d$ in state $s = n+1,\ n+2$. The value of firm equity is also positive if $\varepsilon > 0$, because the shareholders of the representative firm receive $a_H - a_M > 0$ in state $s = n+2$.

The first condition required for the claim in Proposition 9 is that the size of the negative idiosyncratic shock, $a_M - a_L$, is sufficiently large, so that, when $\delta = \varepsilon = 0$, firms choose $\ell = a_M$ and banks choose a fully diversified portfolio in equilibrium. This corresponds to Case (ii) of Proposition 8. The second condition is that $\delta$ and $\varepsilon$ are both positive, but not too large. That is, we are sufficiently close to the case of purely idiosyncratic uncertainty so that firms' and banks' debt level and banks' portfolio are the same as when $\delta = \varepsilon = 0$.

To gain some understanding of why banks choose a positive level of equity under the conditions of Proposition 9, note that firms choose a risky face value of debt, $\ell_j = a_M$, so a firm choosing technology $j$ defaults in state $s = j$. To avoid default in all states, the firm would have to reduce its debt level $\ell_j$ from $a_M$ to $a_L$. Given the relatively small likelihood of state $s = j$, the cost would outweigh the benefits. On the other hand, the introduction of an equity buffer to prevent the default of a diversified bank would be much cheaper. First, the probability that *some* firm whose debt is held by the bank defaults is $n$ times higher than the probability that any given firm defaults. A shock that is unlikely to affect a particular firm is so quite likely to affect a diversified bank. Second, the required equity buffer is much smaller than in the firm's case, because the debt of the defaulting firm is a small fraction of the bank's portfolio. We begin so to see why banks choose to hold a capital buffer against a shock while firms do not.

The table below summarizes the equilibrium values of bank and firm equity in the different cases characterized by Propositions 8 and 9: [15]

| Parameters | Value of bank equity | Value of firm equity |
|---|---|---|
| $0 \leq \delta < \delta^*$, $0 < \varepsilon < \varepsilon^*$ | $> 0$ | $> 0$ |
| $0 < \delta < \delta^*$, $\varepsilon = 0$ | $> 0$ | $= 0$ |
| $\varepsilon + \delta = 1$ | $= 0$ | ?? |
| $\varepsilon + \delta = 0$ | $= 0$ | $> 0$ |

Note that whether firm equity has positive or zero value in the co-monotonic case $\varepsilon + \delta = 1$ depends on parameter values. There are two possible equilibrium outcomes: in one $\ell_j = a_M$ and the value of firm equity is positive, in the other, $\ell_j = a_H$ and the value of firm equity is zero.

The technologies defined by (14) and (15) also allow us to explore the connection between portfolio diversification and the presence of positive bank equity. We noted at the end of

[15] Note that the condition $\left(\frac{n-\lambda_f}{n-1}\right) a_L < a_M$ is maintained here.

Section 4.2 that bank equity cannot have positive value unless banks diversify their portfolios. In other words, bank equity has no role as a buffer unless the bank faces risks that can be partly diversified. At the same time, one can have diversification without a positive value of bank equity, as we saw in case (ii) of Proposition 8, where bank equity has zero value and the bank's portfolio is risk free. But there are other situations where there is a risk of default that could be reduced by diversification, yet banks choose not to diversify and to issue no equity, accepting a positive probability of default.

**Proposition 10** *Assume that firms' technologies satisfy (14), (15) and* $\varepsilon = 0$*. Then there exists* $\delta^{**} > 0$ *such that, for all* $\delta^{**} < \delta < 1$*, any equilibrium has the following properties: each bank lends to firms using a single technology* $j$*; firms using technology* $j$ *have face value of debt* $\ell_j = a_M$*, and bank deposits are* $d = a_M$*. Both the firms using technology* $j$ *and the banks lending to them default in state* $s = j$*.*

The proof follows an argument parallel to the proof of Proposition 9. In the limit, when $\delta = 1$, there is no risk and banks and firms choose $d = \ell = a_M$. For $\delta < 1$ sufficiently close to 1, the upper hemicontinuity of the constrained efficient allocation correspondence implies that Proposition 10 holds. To avoid default, even with diversification, an equity buffer is needed, as we saw in Proposition 9, that is bounded away from zero. As $\delta \to 1$, the cost of the buffer remains bounded away from zero while the benefit converges to zero. When the bank chooses $d = a_M$, on the other hand, the value of diversification is negative: the probability of default is minimized by choosing a simple portfolio and the benefit of this choice increases as $\delta \to 1$. Thus, even if diversification is possible, it is not always optimal. This principle is also illustrated by ACM, who analyse an environment with two i.i.d. technologies and show that, for some parameter values, it is optimal for banks to lend only to firms using one of the technologies, that is, to choose a co-monotonic portfolio. In that case, it will be optimal for the bank equity to have zero value, even though there appears to be scope for diversification. Evidently, the gains from diversification have to be "sufficiently large" before they provide a motive for banks to issue a positive value of equity. In other words, a violation of co-monotonicity is necessary, but not sufficient, for bank equity to have positive value.

# 5 Conclusion

We have presented a classical model of competitive equilibrium in which banks act as intermediaries between productive firms and consumers. Banks and firms raise funds by issuing debt and equity. The Modigliani Miller Theorem does not hold in this environment for two reasons. First, bank deposits offer liquidity services. Second, the use of debt can lead to costly bankruptcy. As a result, the optimal capital structure, for a firm or bank, is determinate in equilibrium. In this context, we have established analogues of the fundamental theorems of welfare economics, showing that equilibria are constrained efficient and that constrained-efficient allocations are decentralizable. Thus, equilibrium capital structures are privately and socially optimal: they maximize the market value of the firm or bank and they are consistent with constrained efficiency. The importance of the general equilibrium theory is that it shows how markets coordinate the choice of capital structures, in the corporate and banking sectors, so that risk and equity capital are optimally allocated between the two sectors.

We have studied an environment in which the laisser-faire equilibrium is constrained efficient and there is no obvious need for capital adequacy regulation. Nonetheless, this may be considered a first step toward a theory of bank capital regulation. In the first place, the characterization of the constrained efficient capital structures gives us some idea of what capital adequacy regulation is trying to achieve. In the second place, the contrast between the ideal conditions under which laisser faire is constrained efficient and the frictions in the real world gives us some idea why market failures occur. In particular, the existence of perfect markets for all possible types of debt and equity is a demanding assumption and unlikely to be satisfied.

We alluded briefly in the introduction to some of the justifications for capital regulation. Asymmetric information is one. Banks are opaque and depositors and equity holders alike may have incomplete information about the risk characteristics of the bank's portfolio. Asymmetric information may give rise to moral hazard in the form of risk shifting and asset substitution. The possibility of government bailouts in the event of default may also encourage excessive risk taking. Externalities, whether pecuniary or real, may give rise to costs that are not internalized by bankers.

Capital requirements may not be a panacea, however. Managers' interests are not aligned with shareholders' interests and, even if they are aligned, it is not clear that the top management of the largest banks is aware of and able to control risk taking by highly incentivized managers at lower levels. The financial crisis provided us with several examples of high level managers, holding large equity stakes in the bank, who were unaware of the dangers facing their banks until the last minute.

We have also ignored a second motive for requiring a large capital buffer: bank capital is part of total loss absorbtion capacity (TLAC), which reduces the need for politically unpopular bailouts. Whether innovations such as TLAC and bail-inable debt will actually put an end to bailouts is not clear, but capital regulation motivated by the desire to avoid bailouts may look quite different from capital regulation motivated by moral hazard in risk taking.

So, there is much to be done in order to develop a satisfactory microfoundation for capital regulation. But the recognition that efficient capital structures in the banking and corporate sectors are interrelated and determined by general equilibrium forces is a first-order requirement for any sensible theory.

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# Appendix

## Proof of Proposition 1

The proof is by contradiction. Suppose, contrary to what we want to prove, that $(\mathbf{c}^*, \mathbf{z}^*, \boldsymbol{\mu}^*, \boldsymbol{\kappa}^*)$ is constrained inefficient. Then there exists a feasible allocation $(\mathbf{c}, \mathbf{z}, \boldsymbol{\mu}, \boldsymbol{\kappa})$ such that $U(\mathbf{c}) > U(\mathbf{c}^*)$. Condition (iii) of the definition of equilibrium implies that $\mathbf{c}$ lies outside the representative consumer's budget set. That is, $\mathbf{q}^* \cdot \mathbf{z} > 1$, where

$$\mathbf{c} = \mathbf{z} \cdot \mathbf{a} = \sum_{h \in H} \left( z_h^d \mathbf{a}_h^d, z_h^e \mathbf{a}_h^e \right)$$

and $H = B \cup F$ is the combined set of types of banks and firms. Now the equilibrium optimality condition for banks implies that

$$q_b^{d*} + q_b^{e*} \leq \mathbf{q}_F^{d*} \cdot \mathbf{x}_b,$$

for any $b \in B$. Then

$$\begin{aligned}
\sum_{b \in B} q_b^{d*} z_b^d + \sum_{b \in B} q_b^{e*} z_b^d &= \sum_{b \in B} \mu_b \left( q_b^{d*} + q_b^{e*} \right) \\
&\leq \sum_{b \in B} \mu_b \mathbf{q}_F^{d*} \cdot \mathbf{x}_b \\
&= \sum_{f \in F} \kappa_f q_f^{d*}
\end{aligned}$$

because attainability requires that $z_b^d = z_b^e = \mu_b$, for any $b = (\mathbf{x}, d) \in B$, and $\sum_{b \in B} \mu_b x_f = \kappa_f$, for any $f \in F$.

Similarly, the equilibrium optimality condition for firms implies that

$$q_f^{d*} + q_f^{e*} \leq 1,$$

for any $f \in F$. But this implies that

$$\begin{aligned}
\mathbf{q}^* \cdot \mathbf{z} &= \sum_{b \in B} q_b^{d*} \mu_b + \sum_{b \in B} q_b^{e*} \mu_b + \sum_{f \in F} q_f^{e*} \kappa_f \\
&\leq \sum_{f \in F} q_f^{d*} \kappa_f + \sum_{f \in F} q_f^{e*} \kappa_f \\
&\leq \sum_{f \in F} \kappa_f = 1.
\end{aligned}$$

This contradicts our initial hypothesis and proves that the equilibrium allocation must be efficient.

## Proof of Proposition 2

Let $C$ denote the set of attainable consumption vectors. Then the supporting hyperplane theorem tells us that there exists a non-negative price vector $\mathbf{p}^*$ such that

$$\mathbf{p}^* \cdot \mathbf{c}^* = \sup \{\mathbf{p}^* \cdot \mathbf{c} : \mathbf{c} \in C\}.$$

Without loss of generality, we normalize prices so that $\mathbf{p}^* \cdot \mathbf{c}^* = 1$. Now define the securities' prices $\mathbf{q}^*$ as follows:

$$\begin{aligned} q_b^{d*} &= \mathbf{p}^* \cdot \left(\mathbf{a}_b^d, \mathbf{0}\right), \\ q_b^{e*} &= \mathbf{p}^* \cdot \left(\mathbf{0}, \mathbf{a}_b^e\right), \\ q_f^{e*} &= \mathbf{p}^* \cdot \left(\mathbf{0}, a_f^e\right), \text{ and} \\ q_f^{d*} &= 1 - q_f^{e*}. \end{aligned}$$

For any $(\mathbf{c}, \boldsymbol{\mu}, \boldsymbol{\kappa})$ satisfying (11a) and (12a),

$$\begin{aligned} 1 &= \mathbf{p}^* \cdot \mathbf{c}^* \\ &\geq \mathbf{p}^* \cdot \mathbf{c} \\ &= \mathbf{p}^* \cdot \left( \sum_{b \in B} \mu_b \left(\mathbf{a}_b^d, \mathbf{a}_h^e\right) + \sum_{f \in F} \kappa_f \left(\mathbf{0}, \mathbf{a}_f^e\right) \right) \\ &= \sum_{b \in B} \mu_b \left(q_b^{d*} + q_b^{e*}\right) + \sum_{f \in F} \kappa_f q_f^{d*} \\ &= \sum_{b \in B} \mu_b \left(q_b^{d*} + q_b^{e*}\right) + \sum_{b \in B} \mu_b \mathbf{x}_b \cdot \mathbf{q}_F^{d*}. \end{aligned}$$

It follows from this inequality that, for any bank $b$, with portfolio $\mathbf{x}$,

$$\begin{aligned} & q_b^{d*} + q_b^{e*} + \mathbf{x} \cdot \mathbf{q}_F^{d*} \leq 1, \\ \Longrightarrow\ & q_b^{d*} + q_b^{e*} + \mathbf{x} \cdot \left(\mathbf{1} - \mathbf{q}_F^{d*}\right) \leq 1, \\ \Longrightarrow\ & q_b^{d*} + q_b^{e*} - \mathbf{x} \cdot \mathbf{q}_F^{e*} \leq 0. \end{aligned}$$

In other words, no bank can earn positive profits. But active firms must earn zero profits in equilibrium, because

$$\begin{aligned} 1 &= \mathbf{p}^* \cdot \mathbf{c}^* \\ &= \mathbf{p}^* \cdot \left( \sum_{b \in B} \mu_b^* \left(\mathbf{a}_b^d, \mathbf{a}_h^e\right) + \sum_{f \in F} \kappa_f^* \left(\mathbf{0}, \mathbf{a}_f^e\right) \right) \\ &= \sum_{b \in B} \mu_b^* \left(q_b^{d*} + q_b^{e*}\right) + \sum_{f \in F} \kappa_f^* q_f^{e*} \\ &= \sum_{b \in B} \mu_b^* \left(q_b^{d*} + q_b^{e*}\right) + 1 - \sum_{f \in F} \kappa_f^* q_f^{d*} \end{aligned}$$

so

$$\sum_{b\in B}\mu_b^*\left(q_b^{d*}+q_b^{e*}\right)-\sum_{f\in F}\kappa_f^* q_f^{d*}=0.$$

Then $\mu_b^*>0$ implies $q_b^{d*}+q_b^{e*}-\mathbf{x}\cdot\mathbf{q}_F^{d*}=0$.

By definition, we have

$$q_f^{d*}+q_f^{e*}=1,$$

so all firms are value maximizing.

Finally, the optimality of the representative consumer's choice follows from the fact that $U(\mathbf{c})>U(\mathbf{c}^*)$ implies that $\mathbf{p}^*\cdot\mathbf{c}>\mathbf{p}^*\cdot\mathbf{c}^*$. Any portfolio $\mathbf{z}$ that generates a consumption bundle $\mathbf{c}$ must therefore have a value greater than 1 because we have defined security prices so that

$$\mathbf{q}\cdot\mathbf{z}=\mathbf{p}\cdot\mathbf{c}.$$

But this implies that $\mathbf{z}$ does not belong to the budget set $\{\mathbf{z}:\mathbf{q}\cdot\mathbf{z}\le 1\}$.

## Proof of Proposition 3

The proof is by contradiction. Suppose, contrary to what we want to prove, that $d\ge\ell$. Then the bank will be bankrupt with probability one and $d(a)=\lambda_f\ell(a)$ for all $a$. Then the maximand is

$$\int_0^\infty\left\{d(a)+\beta\int_{\ell/a}^\infty(\theta a-\ell)f(\theta)\,d\theta\right\}g(a)\,da=$$

$$\int_0^\infty\left\{\lambda_b\ell(a)+\beta\int_{\ell/a}^\infty(\theta a-\ell)f(\theta)\,d\theta\right\}g(a)\,da=$$

$$\int_0^\infty\left\{\lambda_b\left[\int_0^{\ell/a}\lambda_f\theta af(\theta)\,d\theta+(1-F(\ell/a))\,\ell\right]+\beta\int_{\ell/a}^\infty(\theta a-\ell)f(\theta)\,d\theta\right\}g(a)\,da.$$

Consider the effect of reducing the face value of deposits so that $d$ satisfies

$$\frac{(1-\beta)\,\ell}{2-\beta-\lambda_b}<d<\ell, \tag{16}$$

while holding $\ell$ constant. Note that there exists a non-empty set of such values of $d$ because

$$\frac{(1-\beta)\,\ell}{2-\beta-\lambda_b}=\frac{(1-\beta)\,\ell}{1-\beta+1-\lambda_b}<\ell.$$

Then, for large values of $a$, we avoid the bankruptcy costs imposed on the banks. Let $\bar a$ be the smallest value of $a$ for which the banks are solvent: $\ell(\bar a)=d$. Then the gain from reducing $d$ is

$$\int_{\bar a}^\infty\{\beta(\ell(a)-d)+d-\lambda_b\ell(a)\}\,g(a)\,da=\int_{\bar a}^\infty\{(\beta-1)(\ell(a)-d)+(1-\lambda_b)\,\ell(a)\}\,g(a)\,da$$

which must be positive because the inequality (16) implies that the integrand of the right hand expression is positive:

$$\begin{aligned}(\beta-1)(\ell(a)-d)+(1-\lambda_b)\ell(a) &> (\beta-1)(\ell-d)+(1-\lambda_b)d \\ &= (\beta-1)\ell+(2-\beta-\lambda_b)d \\ &> (\beta-1)\ell+(1-\beta)\ell=0.\end{aligned}$$

## Proof of Proposition 5

In state $s$, the holders of equity in bank $b=(\mathbf{x},d)$ receive

$$\max\left\{\sum_{f^*=(\ell^*,j^*)}\chi_{f^*}(s)\,x_{f^*}\min\{\ell^*,A_{f^*s}\}-d,0\right\}$$

units of consumption and the holders of deposits receive

$$\chi_b(s)\min\left\{d,\sum_{f^*=(\ell^*,j^*)}\chi_{f^*}(s)\,x_{f^*}\min\{\ell^*,A_{f^*s}\}\right\}$$

units of consumption, where

$$\chi_{f^*}(s)=\begin{cases}1 & \text{if } \ell^*\leq A_{j^*s}\\ \lambda_{f^*} & \text{if } \ell^*>A_{j^*s}\end{cases}$$

and

$$\chi_b(s)=\begin{cases}1 & \text{if } d\leq\sum_{f^*=(\ell^*,j^*)}\chi_{f^*}(s)\,x_{f^*}\min\{\ell^*,A_{j^*s}\}\\ \lambda_{f^*} & \text{if } d>\sum_{f^*=(\ell^*,j^*)}\chi_{f^*}(s)\,x_{f^*}\min\{\ell^*,A_{j^*s}\}\end{cases}.$$

Let

$$\ell_b=\sum_{f^*=(\ell^*,j^*)}x_{f^*}\ell^*$$

denote the face value of the debt owed to bank $b$ and let

$$\ell_b(s)=\sum_{f^*=(\ell^*,j^*)}\chi_{f^*}(s)\,x_{f^*}\min\{\ell^*,A_{j^*s}\}$$

denote the actual amount repaid to bank $b$ in state $s$.

**Step 1:** *The value of equity of all active banks is positive if and only if* $\ell_{b^*}>d^*$.
The dividends paid by the bank are non-negative in each state because of limited liability. Then it is clear that the value of equity is strictly positive if and only if $\ell_b(s)>d$ in at least one state $s$. Since default is costly, all active firms choose a debt level $\ell^*\leq A_{fS}$ and

all active banks a level of deposits $d^* \leq \ell_{b^*}$. We claim that the value of equity of bank $b^*$ is positive if and only if $\ell_{b^*} > d^*$. To see this, note first that, if $\ell_{b^*} = d^*$, the payment to equity holders (as defined above) is zero in each state and, second, if $\ell_{b^*} > d^*$, the payment must be positive in state $S$ at least, because in that state each firm $f^*$ repays the face value of its debt, the bank receives $\ell_{b^*}$ and the equity holders receive $\ell_{b^*}(S) - d^* = \ell_{b^*} - d^* > 0$.

**Step 2:** *For each $f^*$, there exists a state $s_{f^*}$ such that firm $f^*$ is solvent if and only if $s \geq s_{f^*}$. Similarly, for each $b^*$ there exists a state $s_{b^*}$ such that bank $b^*$ is solvent if and only if $s \geq s_{b^*}$.*
A firm $f^* = (\ell^*, j^*)$ is insolvent in state $s$ if and only if $\ell^* > A_{j^*s}$. Let $s_{f^*}$ be the smallest state such that $f^*$ is solvent. There must be such a state because the firm is solvent at least in state $S$. The fact that $A_{j^*s}$ is increasing in $s$ implies that $f^*$ is solvent if and only if $s \geq s_{f^*}$.

Similarly, we can show that there is a state $s_{b^*}$ such that bank $b^*$ is solvent if and only if $s \geq s_{b^*}$. Let $s_{b^*}$ denote the smallest state in which the bank is solvent. There must be such a state because the bank is solvent at least in state $S$. It is clear that $\ell_{b^*}(s)$ is non-decreasing in $s$ because $A_{j^*s}$ is increasing in $s$ for every $j$ and $\chi_{f^*}(s)$ is non-decreasing in $s$, for every $f^*$. Let

$$d_{b^*}(s) = \chi_{b^*}(s) \min\{d^*, \ell_{b^*}(s)\}$$

denote the payment to deposit holders in state $s$. Then it is clear that $d_{b^*}(s)$ is non-decreasing in $s$ because $\ell_{b^*}(s)$ and $\chi_{b^*}(s)$ are non-decreasing. From this observation it follows that the bank is solvent if and only if $s \geq s_{b^*}$.

**Step 3:** *The face value of deposits satisfies $d^* = \ell_{b^*}(s_{b^*})$.*
To prove this claim, we have to consider two cases. First, suppose that $s_{b^*} = 1$, that is bank $b^*$ is solvent in every state, and $d^* < \ell_{b^*}(1)$.Then increasing $d$ to $d^* + \varepsilon < \ell_{b^*}(1)$, say, will increase the payout to depositors by $\varepsilon$ in every state and reduce the payout to equity holders by the same amount. This increases effective consumption by $(1-\beta)\varepsilon$ in each state, contradicting the constrained efficiency of the equilibrium.

The second possibility is that $s_{b^*} > 1$ and $d^* < \ell_{b^*}(s_{b^*})$. Suppose the bank increases deposits by $\varepsilon > 0$ to $d^* + \varepsilon < \ell_{b^*}(s_{b^*})$. This will not have any effect in states $s < s_b$ because the bank is in default, depositors are receiving $\ell_{b^*}(s) < d^*$ and equity holders are receiving nothing. In states $s \geq s_{b^*}$, on the other hand, the net effect will be an increase in effective consumption of $(1-\beta)\varepsilon$. This again contradicts constrained efficiency.

**Step 4:** *Bank $b$'s equity has no value: $d^* = \ell_{b^*}$.*
Suppose to the contrary that $d^* = \ell_{b^*}(s_{b^*}) < \ell_{b^*}$. Then there must exist at least one firm that is bankrupt in state $s_{b^*}$. Otherwise, the firms' repayment would be $\ell_{b^*}(s_{b^*}) = \ell_{b^*}$.

Suppose that firm $f^* = (\ell^*, j^*)$ is bankrupt in state $s_{b^*}$ and consider the effect of reducing the borrowing of $f^*$ by an amount $\varepsilon > 0$ such that $\ell^* - \varepsilon = A_{j^* s_{b^*}}$. This change has no impact on the viability of the bank in state $s_{b^*}$ because firm $f^*$ is now paying $A_{j^* s_{b^*}}$ in state $s_{b^*}$, instead of $\lambda_{f^*} A_{j^* s_{b^*}}$. So the bank is solvent in states $s \geq s_{b^*}$ as before. Also, the change does have an effect on the solvency of the firm, as it is now solvent in state $s_{b^*}$ and hence in all states $s \geq s_{b^*}$.

Note that none of the payoffs to the debt and equity of the bank or the firm change in states $s < s_{b^*}$. Moreover, there are no changes to the payoff to bank's debt (deposits) in states $s \geq s_{b^*}$ because the bank is solvent in all these states and hence pays $d^*$ to depositors. The changes in payoff affect only the returns to equity in the states $s \geq s_{b^*}$. Consider first the equity of the bank. In states $s \geq s_{b^*}$, the payoff of bank's equity will increase because of the increase in the firm's repayment, that is the change in the payoff of bank's equity is equal to

$$\Delta_{b^*,s} = \begin{cases} A_{j^* s_{b^*}} - \lambda_{f^*} A_{j^* s} & \text{if } s_{b^*} \leq s < s_{f^*} \\ A_{j^* s_{b^*}} - \ell^* & \text{if } s_{f^*} \leq s \end{cases},$$

because the firm will pay $A_{j^* s_{b^*}}$ in each state and was previously paying $\lambda_{f^*} A_{j^* s}$ in states $s_{b^*} \leq s < s_{f^*}$ and $\ell^*$ in states $s_{f^*} \leq s$.

The return to the equity of firm $f^*$ is increased by $A_{j^* s} - A_{j^* s_{b^*}}$ for all $s \geq s_{b^*}$; the change in the firm's equity is so

$$\Delta_{f^*,s} = \begin{cases} A_{j^* s} - A_{j^* s_{b^*}} & \text{if } s_{b^*} \leq s < s_{f^*} \\ \ell^* - A_{j^* s_{b^*}} & \text{if } s_{f^*} \leq s \end{cases}.$$

The transfer between bank equity holders and firm equity holders has no effect on total consumption. The net increase in consumption is the sum of $\Delta_{b^*,s}$ and $\Delta_{f^*,s}$, which is

$$\begin{aligned} \Delta_{b^*,s} + \Delta_{f^*,s} &= \begin{cases} A_{f^* s_{b^*}} - \lambda_{f^*} A_{j^* s} + A_{j^* s} - A_{j^* s_{b^*}} & \text{if } s_{b^*} \leq s < s_{f^*} \\ A_{j^* s_{b^*}} - \ell^* + \ell^* - A_{j^* s_{b^*}} & \text{if } s_{f^*} \leq s \end{cases} \\ &= \begin{cases} A_{j^* s} - \lambda_{f^*} A_{j^* s} & \text{if } s_{b^*} \leq s < s_{f^*} \\ 0 & \text{if } s_{f^*} \leq s \end{cases}. \end{aligned}$$

Thus, the net gain for equity holders is the saving in default costs $(1 - \lambda_{f^*}) A_{j^* s}$ in the states $s_{b^*} \leq s < s_{f^*}$. The possibility of such a gain contradicts the constrained efficiency of the equilibrium.

This completes the proof that bank equity has no value.

## Proof of Proposition 7.

**Banks holding risk free debt** In the case of a bank holding only the debt of firms with $\ell_j = a_L$, the portfolio $\mathbf{x}$ is indeterminate subject to the constraint $\sum_j x_j^L = 1$. The optimal capital structure for the bank is to issue the maximum amount of deposits, $d = a_L$. The expected utility generated by the bank and the firms whose debt it holds will be

$$\begin{aligned} & d + \varepsilon\beta(a_H - d) + \delta\beta(a_M - d) + (1-\delta-\varepsilon)\beta\left(\frac{n-1}{n}a_M + \frac{1}{n}a_L - d\right) \\ = \; & a_L + \varepsilon\beta(a_H - a_L) + \delta\beta(a_M - a_L) + (1-\delta-\varepsilon)\beta\left(\frac{n-1}{n}a_M - \frac{n-1}{n}a_L\right) \\ = \; & a_L + \varepsilon\beta(a_H - a_L) + \left(\delta + (1-\delta-\varepsilon)\frac{n-1}{n}\right)\beta(a_M - a_L). \end{aligned}$$

**Banks holding safe and risky debt** Now suppose that the bank lends to a mixture of safe and risky firms. We can focus here on the loans made by the bank to firms with technology $j$ but possibly different capital structures. Suppose that $\gamma$ units of capital are invested in safe firms with $\ell_j = a_L$ and $1-\gamma$ units of capital are invested in risky or very risky firms, that is, firms that have a capital structure $\ell_j \in \{a_H, a_M\}$. There is no need to distinguish safe firms according to the technology they use: from the point of view of banks and shareholders, who hold their debt and equity, they are identical. Let $x_j^H$ and $x_j^L$ denote the fraction of $1-\gamma$ invested in firms with technology $j$ and $\ell_j$ equal to $a_H$ and $a_M$, respectively. Note that $\sum_{j=1}^n a_j^H + a_j^M = 1$ and that the amounts invested in firms with technology $j$ and $\ell_j$ equal to $a_H$ and $a_M$ are $a_j^H(1-\gamma)$ and $a_j^M(1-\gamma)$, respectively.

Suppose the bank chooses a level of deposits $d$. Since the safe banks pay $a_L$ for sure, the bank will fail if and only if payment from the risky banks is less than $d - \gamma a_L$. Now suppose that we split the bank into two banks, one of which funds safe firms and the other funds risky firms. The safe bank invests one unit in safe firms and issues deposits $d^S = a_L$ and the risky bank invests one unit in a portfolio $\left\{x_j^H, x_j^M\right\}_{j=1}^n$ of risky firms and issues deposits $d^R$. The expected utility from the safe bank is denoted by $U^S$ and the expected utility from the risky bank is denoted by $U^R$. What is the difference between these two banks and the combined bank we started with? Note that the risky bank will default if and only if the unified bank defaults with a positive probability. If there is no probability of default, there is no difference in the expected utility generated by the two structures. On the other hand, if there is a positive probability of default, the separated banks will generate a higher expected utility, because the safe bank does not default whereas the combined bank does default in some states. In fact, the gain in expected utility by separating the banks is precisely the probability of default muliplied by the default cost $(1-\lambda_b)\,d_b^S$.

Thus, either there is no gain from mixing safe and risky debt in the banks portfolio or, if the mixture of safe and risky debt causes the bank to default with positive probabilty, there is a loss.

**Safe banks holding risky debt** Now suppose that a bank chooses a portfolio $\mathbf{x} = \left\{\left(x_j^H, x_j^M, 0\right)\right\}_{j=1}^n$ where $x_j^H$ is the measure of firms of type $j$ with $\ell_j = a_H$, $x_j^M$ is the measure of firms of type $j$ with $\ell_j = a_M$, and we assume that no firms with $\ell_j = a_L$ are included. The portfolio $\mathbf{x}$ has no impact in states $s = n+1, n+2$ because all technologies have identical payoffs in these states. Now consider the states $s = 1, ..., n$ and let $\rho_j$ denote the repayment of all firms when type $j$ has productivity $a_L$. Then

$$\rho_j = \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H + x_j^M\right) \lambda_f a_L.$$

Without essential loss of generality, we can order the types of firms so that $\rho_j \leq \rho_{j+1}$ for $j = 1, ..., n-1$. The bank wants to maximize the face value of deposits subject to the no-default constraint $d \leq \rho_1$. To do that, it must choose a portfolio $\mathbf{x}$ such that $\left(x_j^H, x_j^M\right) = \left(0, \frac{1}{n}\right)$ for all $j = 1, ..., n$, that is, a simple portfolio. Having done so, the value of deposits it can safely issue is

$$d = \frac{n-1}{n} a_M + \frac{1}{n} \lambda_f a_L.$$

The expected utility generated by the bank and the firms whose debt it holds will be

$$\begin{aligned} & d + \beta\left(1 - \delta - \varepsilon\right)\left(\frac{n-1}{n} a_M + \frac{1}{n}\lambda_f a_L - d\right) + \beta\delta\left(a_M - d\right) + \beta\varepsilon\left(a_H - d\right) \\ = \;& d + \beta\delta\left(a_M - d\right) + \beta\varepsilon\left(a_H - d\right). \end{aligned}$$

**Risky banks holding risky debt** We split the analysis in two parts, considering first that case where $d \leq a_M$ and, second, the case where $d > a_M$.

**i)** Suppose that there is a positive probability of the bank defaulting, but that $d \leq a_M$. This means that default only occurs in states $s = 1, ..., n$. As before, the portfolio $\mathbf{x}$ is irrelevant in states $s = n+1, n+2$ so we restrict attention to the states $s = 1, ..., n$. With our usual convention that $\rho_j \leq \rho_{j+1}$, there exists a technology $k$ such that, $d > \rho_j$ for $j = 1, ..., k$ and $d \leq \rho_j$ for $j = k+1, ..., n$. (The bank will never choose to default with probability one). The expected utility of the bank's depositors and shareholders will be

$$\begin{aligned} \frac{1}{n}\lambda_b \sum_{j=1}^k \rho_j + \frac{n-k}{n} d + \frac{1}{n}\beta \sum_{j=k+1}^n \left(\rho_j - d\right) \;&\leq\; \frac{1}{n}\lambda_b \sum_{j=1}^k \rho_j + \frac{n-k}{n} d + \frac{1}{n} \sum_{j=k+1}^n \left(\rho_j - d\right) \\ &=\; \frac{1}{n}\lambda_b \sum_{j=1}^k \rho_j + \frac{1}{n} \sum_{j=k+1}^n \rho_j, \end{aligned}$$

because $\beta < 1$. Now

$$\begin{aligned}
\frac{1}{n}\lambda_b \sum_{j=1}^{k} \rho_j &= \frac{1}{n}\lambda_b \sum_{j=1}^{k} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
&= \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
&\quad + \frac{1}{n}\lambda_b \sum_{j=2}^{k} \left(x_j^H \lambda_f + x_j^M\right) a_M - \frac{1}{n}\lambda_b \sum_{j=2}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \\
&\quad + \frac{1}{n}\lambda_b \sum_{j=2}^{k} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H + x_j^M\right) \lambda_f a_L \right)
\end{aligned}$$

$$\begin{aligned}
\leq\ & \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
& + \frac{1}{n} \sum_{j=2}^{k} \left(x_j^H \lambda_f + x_j^M\right) a_M - \frac{1}{n} \sum_{j=2}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \\
& + \frac{1}{n} \sum_{j=2}^{k} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H \lambda_f + x_j^M\right) \lambda_f a_L \right),
\end{aligned}$$

because $\lambda_b < 1$ and $\left(x_j^H \lambda_f + x_j^M\right) a_M \geq \left(x_j^H + x_j^M\right) \lambda_f a_L$,

$$\begin{aligned}
=\ & \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
& + \frac{1}{n} \sum_{j=2}^{k} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H \lambda_f + x_j^M\right) a_M \right) \\
=\ & \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
& + \frac{k-1}{n} \sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M .
\end{aligned}$$

Substituting this upper bound into the expression for expected utility, we obtain the inequality

$$
\begin{aligned}
&\frac{1}{n}\lambda_b \sum_{j=1}^{k} \rho_j + \frac{1}{n} \sum_{j=k+1}^{n} \rho_j \\
&\leq \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) + \frac{k-1}{n} \sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M \\
&+ \frac{1}{n} \sum_{j=k+1}^{n} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H + x_j^M\right) \lambda_f a_L \right)
\end{aligned}
$$

$$
\begin{aligned}
&\leq \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) + (1-\lambda_b) \frac{1}{n} \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M \\
&- (1-\lambda_b) \frac{1}{n} \sum_{j=k+1}^{n} \left(x_j^H + x_j^M\right) \lambda_f a_L + \frac{k-1}{n} \sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M \\
&+ \frac{1}{n} \sum_{j=k+1}^{n} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
&= \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H + x_j^M\right) \lambda_f a_L + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) + \frac{k-1}{n} \sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M \\
&+ \frac{1}{n} \sum_{j=k+1}^{n} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H \lambda_f + x_j^M\right) a_M \right)
\end{aligned}
$$

because $\lambda_b < 1$ and $\sum_{j=k+1}^{n} \left(x_j^H + x_j^M\right) \lambda_f a_L < \sum_{j=k+1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M$. So

$$
\begin{aligned}
&\frac{1}{n}\lambda_b \sum_{j=1}^{k} \rho_j + \frac{1}{n} \sum_{j=k+1}^{n} \rho_j \\
&\leq \frac{1}{n}\lambda_b \left( \sum_{j=k+1}^{n} \left(x_j^H + x_j^M\right) \lambda_f a_L + \sum_{j=1}^{k} \left(x_j^H + x_j^M\right) \lambda_f a_L \right) \\
&+ \frac{k-1}{n} \sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) a_M + \frac{1}{n} \sum_{j=k+1}^{n} \left( \sum_{i \neq j} \left(x_i^H \lambda_f + x_i^M\right) a_M + \left(x_j^H \lambda_f + x_j^M\right) a_M \right) \\
&\leq \frac{1}{n}\lambda_b \lambda_f a_L + \frac{k-1}{n} a_M + \frac{n-k}{n} a_M = \frac{1}{n}\lambda_b \lambda_f a_L + \frac{n-1}{n} a_M,
\end{aligned}
$$

because $\sum_{j=1}^{n} \left(x_j^H \lambda_f + x_j^M\right) \leq \sum_{j=1}^{n} \left(x_j^H + x_j^M\right) = 1$. But the last expression in this series of inequalities is the representative consumer's expected utility when the bank lends only to firms that use a single technology $j$ and choose the capital structure $\ell = a_M$ and the level of deposits is $d = a_M$. It is easy to check that for any other portfolio and face value of deposits one of the inequalities is strict, so this is the unique policy that maximizes expected utility in the states $s = 1, ..., n$ when $d \leq a_M$ and the probability of default is positive.

Now let us check that this policy is optimal in the states $s = n+1, n+2$. The expected utility in states $s = n+1, n+2$ is

$$\frac{\delta}{\delta+\varepsilon} a_M + \frac{\varepsilon}{\delta+\varepsilon} \left( a_M + \beta \left( a_H - a_M \right) \right).$$

For any other portfolio $\left\{ \left( x_j^H, x_j^M, 0 \right) \right\}_{j=1}^n$, the bank will be in default in state $s = n+1$ if $\sum_{j=1}^n x_j^H > 0$ and the expected utility in states $s = n+1, n+2$ is

$$\frac{\delta}{\delta+\varepsilon} \lambda_b \left( \sum_{j=1}^n \left( x_j^H \lambda_f + x_j^M \right) a_M \right) + \frac{\varepsilon}{\delta+\varepsilon} \left( a_M + \beta \left( a_H - a_M \right) \right) <$$

$$\frac{\delta}{\delta+\varepsilon} a_M + \frac{\varepsilon}{\delta+\varepsilon} \left( a_M + \beta \left( a_H - a_M \right) \right).$$

If $\sum_{j=1}^n x_j^H = 0$, the bank is not in default in either state and the payoff is

$$\frac{\delta}{\delta+\varepsilon} \left( \sum_{j=1}^n x_j^M a_M \right) + \frac{\varepsilon}{\delta+\varepsilon} \left( a_M + \beta \left( a_H - a_M \right) \right) =$$

$$\frac{\delta}{\delta+\varepsilon} a_M + \frac{\varepsilon}{\delta+\varepsilon} \left( a_M + \beta \left( a_H - a_M \right) \right)$$

Thus, the unique optimal policy is to set $d = a_M$ and $x_j^M = 1$ for some $j$ as long as $d \leq a_M$.

**ii)** Now consider the case in which $d > a_M$. In that case, the bank always defaults states $s = 1, ..., n+1$. In states $s = 1, ..., n$, the expected utility will be

$$\frac{1}{n} \sum_{j=1}^n \lambda_b \left( \sum_{i \neq j} \left( x_i^H \lambda_f + x_i^M \right) a_M + \left( x_j^H + x_j^M \right) \lambda_f a_L \right)$$

and in states $s = n+1, n+2$ it will be

$$\frac{\delta}{\delta+\varepsilon} \lambda_b \sum_{j=1}^n \left( x_j^H \lambda_f + x_j^M \right) a_M + \frac{\varepsilon}{\delta+\varepsilon} \left( d + \beta \left( a_H - d \right) \right).$$

The choice of $d$ will be the maximum that allows the bank to remain solvent in state $s = n+2$, that is, $d = \sum_{j=1}^n \left( x_j^H a_H + x_j^M a_M \right)$. Letting $x^H = \sum_{j=1}^n x_j^H$ and $x^M = \sum_{j=1}^n x_j^M$, we can rewrite the expected utility as

$$\frac{\delta}{\delta+\varepsilon} \lambda_b \left( x^H \lambda_f + x^M \right) a_M + \frac{\varepsilon}{\delta+\varepsilon} \left( d + \beta \left( a_H - d_b \right) \right)$$

in states $s = n+1, n+2$ and

$$\lambda_b \left( \frac{n-1}{n} \left( x^H \lambda_f + x^M \right) a_M + \frac{1}{n} \lambda_f a_M \right)$$

in states $s = 1, ..., n$. This expression is linear in $\left(x^H, x^M\right)$, so at least one of the extreme points $\left(x^H, x^M\right) = (0,1)$ or $\left(x^H, x^M\right) = (1,0)$ must be an optimum. Since we assume that $d > a_M$, this case is only observed if $x^H = 1$.

## Proof of Proposition 8.

As shown in the text, if the representative bank lends to safe firms ($\ell = a_L$) the expected utility is

$$a_L + \frac{n-1}{n} \beta \left( a_M - a_L \right).$$

If the representative bank issues safe debt (deposits) and lends to risky firms ($\ell = a_M$), on the other hand, the expected utility is

$$\frac{n-1}{n} a_M + \frac{1}{n} \lambda_f a_L + \frac{n-1}{n} \frac{1}{n} \beta \left( a_M - \lambda_f a_L \right).$$

It is strictly optimal to issue safe debt and lend to risky firms if

$$a_L + \frac{n-1}{n} \beta \left( a_M - a_L \right) < \frac{n-1}{n} a_M + \frac{1}{n} \lambda_f a_L + \frac{n-1}{n} \frac{1}{n} \beta \left( a_M - \lambda_f a_L \right).$$

Multiplying by $\frac{n}{n-1}$ yields

$$\frac{n}{n-1} a_L + \beta \left( a_M - a_L \right) < a_M + \frac{n}{n-1} \lambda_f a_L + \frac{1}{n} \beta \left( a_M - \lambda_f a_L \right)$$

and collecting like terms gives us

$$\frac{n}{n-1} \left( 1 - \frac{1}{n} \lambda_f \right) a_L - \beta \left( 1 - \frac{1}{n} \lambda_f \right) a_L < \left( 1 - \frac{n-1}{n} \beta \right) a_M$$

This can be rewritten as

$$\left( \frac{n}{n-1} - \beta \right) \left( 1 - \frac{1}{n} \lambda_f \right) a_L < \left( 1 - \frac{n-1}{n} \beta \right) a_M$$

which is equivalent to

$$\left( \frac{n - \beta (n-1)}{n-1} \right) \left( 1 - \frac{1}{n} \lambda_f \right) a_L < \left( \frac{n - (n-1) \beta}{n} \right) a_M$$

or

$$\left( \frac{n - \lambda_f}{n-1} \right) a_L < a_M.$$

## Proof of Proposition 9

The argument in the proof of Proposition 7 left us with the following candidates for an optimal bank policy.

1. The bank invests in firms with $\ell = a_L$ and $d = a_L$. The firms' types are irrelevant because firm debt is risk free. The expected utility in equilibrium is

$$a_L + \beta\left((1-\delta-\varepsilon)\left(\frac{n-1}{n}a_M + \frac{1}{n}a_L\right) + \delta a_M + \varepsilon a_H - a_L\right).$$

2a. The bank invests in firms with $\ell = a_M$. The portfolio is defined by $x_j^M = \frac{1}{n}$ for all $j$ and the face value of deposits is $d = \frac{n-1}{n}a_M + \frac{1}{n}\lambda_f a_L$. The expected utility in equilibrium is

$$\frac{n-1}{n}a_M + \frac{1}{n}\lambda_f a_L + \beta\left(\delta+\varepsilon\right)\left(\frac{\delta}{\delta+\varepsilon}a_M + \frac{\varepsilon}{\delta+\varepsilon}a_H - \frac{n-1}{n}a_M - \frac{1}{n}\lambda_f a_L\right)$$

2b. The bank invests in firms with $\ell = a_M$. The portfolio is defined by $x_j^M = 1$ for some $j$ and the face value of deposits is $d = a_M$. The expected utility in equilibrium is

$$(1-\delta-\varepsilon)\lambda_b\left(\frac{n-1}{n}a_M + \frac{1}{n}\lambda_f a_L +\right) + (\delta+\varepsilon)a_M + \varepsilon\beta\left(a_H - a_M\right).$$

3. The bank invests in firms with $\ell = a_H$. The portfolio is defined by $x^H = 1$ (the distribution over $j$ is irrelevant) and the face value of deposits is $d = a_H$. The expected utility in equilibrium is

$$\varepsilon a_H + \lambda_b\left(\delta\lambda_f a_M + (1-\delta-\varepsilon)\left(\frac{n-1}{n}\lambda_f a_M + \frac{1}{n}\lambda_f a_L\right)\right).$$

Suppose that $\delta$ and $\varepsilon$ converge to zero. The expected utilities in the different cases converge to

$$a_L + \beta\left(\frac{n-1}{n}a_M - \frac{n-1}{n}a_L\right), \qquad \text{(Case 1)}$$

$$\frac{n-1}{n}a_M + \frac{1}{n}\lambda_f a_L, \qquad \text{(Case 2a)}$$

$$\lambda_b\left(\frac{n-1}{n}a_M + \frac{1}{n}\lambda_f a_L +\right), \qquad \text{(Case 2b)}$$

and

$$\lambda_b \lambda_f \left( \frac{n-1}{n} a_M + \frac{1}{n} a_L \right), \qquad \text{(Case 3)}$$

respectively. Finally, Proposition 9 guarantees that Case 2a dominates Case 1. Thus, Case 2a dominates all other cases for values of $\delta$ and $\varepsilon$ sufficiently close to zero.