Poncelet coefficients for dry sand

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Poncelet Framework

Beyond an impact transient, penetration in sand can be well described by a Poncelet Equation

\[ F = \sigma A = \frac{1}{2} CA \rho V^2 + AR \]

C plays the role of drag coefficient. It is due mainly to inertia. It can have a small velocity dependence. R plays the role of frictional resistance (aka bearing stress). R increases with depth.
Typical velocity decay from Poncelet

Typical Poncelet penetration curve: $dV/dx$ proportional to $V$ until $R$ term dominates, then $dV/dx$ goes to negative infinity as $V$ goes to zero.

$(15\text{mm dia steel ball hitting sand at } 100\text{ m/s})$
Characteristic Velocities

- $V^*$ = velocity above which sand comminutes

- $V_t$ = velocity below which a drag-only version of Poncelet equation is not adequate.

![Graph showing velocity and time records for dense sand](image)

- Impact velocity = 79.7 m/s

- $C = ma/\rho A^2$

- Velocity record
- Time record
- Drag fit
For Dry Sand

We want to determine

• Values of Poncelet drag, C
• Values of characteristic velocities $v^*$ and $v_t$

Source data for hemispherical noses
(G) Glößner et al (2017)
(O) Omidvar et al (2015)
(P) Peden et al (2014)
(B) Bless et al (2009)
Approach to data analysis

- Compute $\frac{M(dv/dt)}{\rho A v^2}$
- For drag-dominated Poncelet regime, this $= C$.
- Plot this versus penetration velocity.
- Determine $C(v)$ and critical velocities.
Glöbner et al Data

30-mm dia steel rods, silica sand, on board accelerometer. To 360 m/s.

Loose: $v^*$ about 100 m/s. $v_c$ about 45 m/s. $C=1$ satisfactory for most purposes. Dense: $C = 0.6$ for $v > 110$ m/s. For $v^* > v > v_t$ $C$ is about 0.75. $v_t$ about 80 m/s
Omidvar et al data

10 mm dia hollow steel rods, \((x,t)\) data, Ottawa sand, to 80 m/s.

Loose:
- Impact velocity = 88.9 m/s
- \(C \approx 0.7\)

Dense:
- Impact velocity = 79.7 m/s
- \(V_t \approx 35\) m/s
Peden et al data

14-mm dia steel spheres, Ottawa sand, PDV, to 300 m/s

Loose sand
Impact velocity=297.4 m/s
C about 0.7

Dense sand
Impact velocity=299.2 m/s
C about 1
C about 1.1
V* about 150 m/s
V_t about 50 m/s
Bless et al

- 5 mm dia fin-stabilized W rods, loose Ottawa sand, to 615 m/s
- \( V^* \) about 80 m/s (by inspection)
- \( C \) about 0.8 for \( v > v^* \)
Characteristic velocities summary

<table>
<thead>
<tr>
<th>Density</th>
<th>$V^*$ (m/s)</th>
<th>$v_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>G 100 B80</td>
<td>G45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P50</td>
</tr>
<tr>
<td>Dense</td>
<td>G100 P100</td>
<td>G80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P35</td>
</tr>
</tbody>
</table>

The value of $v^*$ is between 80 and 100 m/s for silica sand both loose and dense. Values of $v_t$ show quite a lot of scatter. This may depend on grain size and diameter.
## C values summary

<table>
<thead>
<tr>
<th>Regime</th>
<th>Loose</th>
<th>Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>V&gt;v*</td>
<td>G 1.0</td>
<td>G 0.6</td>
</tr>
<tr>
<td></td>
<td>P 0.7</td>
<td>P 1.0</td>
</tr>
<tr>
<td></td>
<td>B 0.8</td>
<td></td>
</tr>
<tr>
<td>V&lt;v*</td>
<td>G 1.0</td>
<td>G 0.75</td>
</tr>
<tr>
<td></td>
<td>O 0.7</td>
<td>O 1.1</td>
</tr>
<tr>
<td></td>
<td>P 0.7</td>
<td>P 1.0</td>
</tr>
</tbody>
</table>

Poncelet constant drag formula is reasonably accurate. C values generally ±0.1. Generally C increases with density. V* has little effect on C. For loose sand C is less at larger scales.