Expected Stock Returns and the Correlation Risk Premium

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Motivation

- Simple question: Is the aggregate market return predictable?
  - Predictable out-of-sample?
- Narrow focus: Forward-looking Variables - Option-Implied
  - Is there a theoretical link of the Variance risk premium (VRP) and correlation risk premium (CRP) to the equity risk premium?
- Redundancy in variables or complementary?
  - CRP is a part of the market VRP!

Contribution and Major Results

1. Derive a market risk premium decomposition, including compensation for variance and correlation risk using a GE model.
2. A new methodology for estimating variance and correlation betas.
   - Use contemporaneous instead of lagged predictive regression.
   - Use shocks to integrated variance/correlation under the $\mathbb{Q}$ measure.
3. Predictability using VRP and CRP—in-sample and **out-of-sample**.
   - VRP predicts best at quarterly horizon; then its performance weakens.
   - CRP weaker at short horizons, but predicts returns up to a year.
4. Study the sources for correlation risk premium.

Model Walkthrough

1. Use N dividend trees adding up to an aggregate consumption process
2. Variance of / and correlation between trees are stochastic
3. $\Rightarrow$ Aggregate consumption variance is stochastic (2 components)
4. Solve for a GE assuming complete markets and EZ rep agent
5. Derive market VRP and CRP between dividend claims
6. Express the ERP as a function of VRP and CRP

Model Assumptions I

- A large number of individual Lucas (1978) trees
  - : Variance is stochastic (a square root process)
  - : Correlation between trees is stochastic:

$$\frac{dD_{i,t} \times dD_{j,t}}{\sqrt{(dD_{i,t})^2 \times (dD_{j,t})^2}} = \rho_{ij,t} dt.$$

  - : Whereas all correlations are driven by the same state variable ρ_t

Model Assumptions II

- Adding up the trees produces the aggregate consumption process with stochastic variance:

$$
\begin{cases}
\frac{dC_t}{C_t} & = \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t} \\
dV_t & = \kappa_1(\bar{V} - V_t)dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \sigma_\rho d\rho_t \\
d\rho_t & = \kappa_2(\bar{\rho} - \rho_t)dt + \sigma_2 \nu(\rho_t) dB_{\rho,t},
\end{cases}
$$

- Two-component variance:
  1. Average individual variance shock: dB_V
  2. Dividend correlation shock: $d\rho$
- *Preferences*
  - Representative investor with continuous-time, recursive preferences.
  - $J_t = E_t\left[\int_t^T f(C_s, J_s)ds\right]$

Model: Solving for Equilibrium I

- Obtain the pricing kernel dynamics:

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dB_{c,t} - \lambda_2 dB_{V,t} - \lambda_3 dB_{\rho,t},$$

where

$$\begin{cases} \lambda_1 & = \gamma \delta_c \sqrt{V_t} \\ \lambda_2 & = -\frac{1-\gamma\psi}{1-\gamma} A_1 \sigma_1 \sqrt{V_t} \\ \lambda_3 & = -\frac{1-\gamma\psi}{1-\gamma} \left(A_1 \sigma_\rho + A_2 \sigma_2 \right) \sqrt{\rho_t} \end{cases}$$

: λ_2—price of average individual variance risk

: λ_3—price of correlation risk

Model: Market Dynamics and Risk Premiums - VRP

- Solve for the dynamics of the market

$$\frac{dW_t}{W_t} = \zeta_W dt + \delta_c \sqrt{V_t} dB_{c,t} - A_{1a} dV_t - A_{2a} d\rho_t,$$

  - Driven by shocks to consumption, its variance and dividend correlation

- Derive the instantaneous variance V_W of the aggregate market and along these lines the VRP.

Model: Market Dynamics and Risk Premiums - CRP

What about the *CRP*?

1. Given the individual dividend process

$$\frac{dD_{i,t}}{D_{i,t}} = \mu_D dt + \sigma_D \sqrt{V_{i,t}} dB_{D_i,t} + \sigma_{DC} \sqrt{V_t} dB_{c,t}$$

   one can exploit the pricing kernel (and other identities / manipulations) to get the dynamics of a stock S.

2. This allows to calculate the variance of an individual stock, the correlation dynamics between any two stocks, and the *CRP*.

Model: Market Dynamics and Risk Premiums - ERP

- One can write the market risk premium as the sum of the three risk components:

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right]/dt - r_{f,t} = \lambda_1 \delta_c \sqrt{V_t} + A_{1z} VRP_t + A_{2z} CRP_t,$$

  - Both risk premiums are directly observable from options data!
  - Decomposition is similar to Bollerslev, Tauchen, and Zhou (2009)
    We have one additional component – CRP

- One can rewrite the market process in terms of the market variance and the correlation between stocks

$$\frac{dW_t}{W_t} = \zeta_W' dt + \delta_c \sqrt{V_t} dB_{c,t} - \mathbf{A_{1z}} dV_{W,t} - \mathbf{A_{2z}} d\rho_{S,t} - A_{3z} dV_{i,t},$$

In-Sample Predictability

$$r_{s\to s+\tau_r} = a + b\, VRP(s, s+\tau_r) + c\, CRP(s, s+\tau_r) + \epsilon,$$

	30d-Ret,		91d-Ret		182d-Ret		273d-Ret		365d-Ret	
VRP	-	0.322	-	0.562	-	-0.304	-	-0.599	-	-0.740
	-	**0.004**	-	**0.002**	-	0.553	-	0.380	-	0.253
CRP	0.076	-	0.254	-	0.381	-	0.588	-	0.559	-
	0.027	-	0.002	-	**0.051**	-	**0.067**	-	0.150	-
R^2	2.48	**6.90**	7.26	**5.08**	**6.90**	0.15	**9.87**	0.81	5.43	0.85

- In-sample predictability for the **VRP at short horizon**.
  (Bollerslev, Tauchen, and Zhou (2009))
- In-sample predictability for the **CRP at longer horizon**.
- Indication for non-redundant information in the two variables.

OOS Predictability - Traditional Approach

- Estimate the parameters of the pricing equation by running a rolling-window (last 3 years) regression for t:

$$r_{s \to s+\tau_r} = \alpha + \beta_{VRP}\, VRP(s, s+\tau_r) + \beta_{CRP}\, CRP(s, s+\tau_r),$$

where $s + \tau_r \leq t$, and regressions are estimated at s.

- Regress historical market excess returns on lagged regressors,

New Approach - Theoretical Foundation

- We estimate the exposures using a contemporaneous regression:

$$\frac{dW_t}{W_t} = \zeta'_W dt + \delta_c \sqrt{V_t} dB_{c,t} - \mathbf{A_{1z}} dV_{W,t} - \mathbf{A_{2z}} d\rho_{S,t} - A_{3z} dV_{i,t},$$

relying on shocks to variance and correlation.

- **Innovation**:
Quadratic co-variation depends **only** on stochastic parts
  - ..which do not change by the change of measure (Girsanov).

$$\frac{dW_t^{\mathbb{Q}}}{W_t} = \zeta''_W dt + \delta_c \sqrt{V_t} dB^{\mathbb{Q}}_{c,t} - \mathbf{A_{1z}} dV^{\mathbb{Q}}_{W,t} - \mathbf{A_{2z}} d\rho^{\mathbb{Q}}_{S,t} - A_{3z} dV^{\mathbb{Q}}_{i,t},$$

$\rightarrow$ Mix realized returns and shocks to option-implied variables!

New Approach - Implementation I

- Estimate the betas using shocks to IV and IC:

$$r_{s+1} - r_{f,s} = \alpha + \beta_{t,\Delta IV} \Delta IV(s+1, T) + \beta_{t,\Delta IC} \Delta IC(s+1, T)$$

- Daily increments in forward-looking moments (last year) —up to now.
- **Important:** Adjust the betas for difference in variances of regressors and predictors:

$$\beta_{t,VRP} = \beta_{t,\Delta IV} \times \frac{\text{Vol}\left(\Delta IV(t, t+\tau)\right)}{\text{Vol}\left(VRP(t, t+\tau)\right)}.$$

- Form a market excess return forecast for horizon τ_r using
  - resulting betas ($\beta_{t,VRP}$ and $\beta_{t,CRP}$), and
  - time-t observable variables $VRP(t, t+\tau_r)$ and $CRP(t, t+\tau_r)$,
  - no intercept is included in the forecast

OOS - Performance Criteria

- Compare 4 models: historical mean, VRP, CRP, and VRP+CRP.
- Performance Criteria
  - OOS R^2

$$R^2_{j,\tau_r} = 1 - \frac{MSE_{j,\tau_r}}{MSE_{1,\tau_r}}, \text{ where } MSE_{j,\tau_r} = \frac{1}{N} e_{j,\tau_r}^\top \times e_{j,\tau_r}.$$

  - Diebold and Mariano (1995) loss function:

$$\delta_{j,\tau_r} = MSE_{j,\tau_r} - MSE_{1,\tau_r}.$$

  - Gain in the Certainty Equivalent of a mean-variance (log) investor:

$$\Delta CE_{j,\tau_r} = CE_{j,\tau_r} - CE_{1,\tau_r}, \text{ where } CE_{j,\tau_r} = E[r^{MV}_{j,\tau_r}] - \frac{\gamma}{2}\sigma^2(r^{MV}_{j,\tau_r}).$$

  - Evaluate significance by moving-block bootstrap by Künsch (1989).

OOS Predictability: Traditional Approach II

	R^2_{j,τ_r}		
Days	VRP	CRP	VRP+CRP
30	-0.438	-0.014	-0.679
	0.000	0.000	0.000
91	-0.938	-0.022	-0.841
	0.000	0.000	0.000
182	-1.667	-0.177	-0.674
	0.000	0.000	0.000
273	-0.453	-0.325	-1.223
	0.000	0.000	0.000
365	-1.628	-0.226	-3.687
	0.000	0.000	0.000

- Practically, no predictability (adj. $R^2 < 0$, $\delta > 0$).
- Common finding for many variables (Goyal and Welch (2008)).

OOS - Contemp. Betas

	R^2_{j,τ_r}		
Days	VRP	CRP	VRP+CRP
30	**0.094**	0.025	0.096
	0.000	0.000	0.000
91	**0.103**	0.081	0.104
	0.000	0.000	0.000
182	0.062	0.067	0.063
	0.000	0.000	0.001
273	0.039	**0.079**	0.032
	0.064	0.000	0.144
365	0.037	**0.070**	0.024
	0.091	0.001	0.210

- Strong out-of-sample predictability for **VRP at short horizon**.
- Strong out-of-sample predictability for **CRP even at longer horizons**.

OOS - Intuition I

Traditional Approach

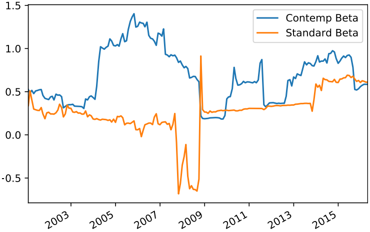
- Very sensitive to outliers
- Delay of data in out-of-sample predictions.
- Example: Predict quarterly returns.
  - In the beta estimation, the last observation is $r_{t-3m\rightarrow t}$, and ...
  - ... the most up-to-date RHS variables are 3 months old!

Contemporaneous Betas Approach

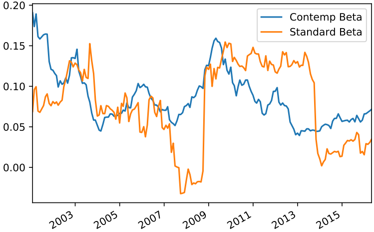
- Most recent RHS variables are from the day of the forecast.
- "High-Frequency" daily data (over the last year).

OOS - Intuition II

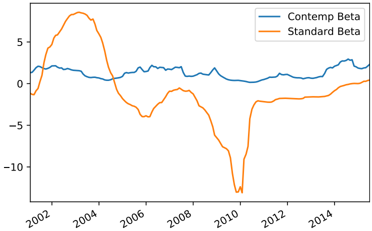
(a) 30-day VRP Betas



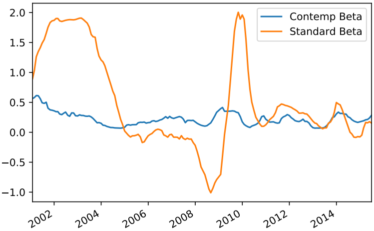
(b) 30-day CRP Betas



(g) 365-day VRP Betas



(h) 365-day CRP Betas



Conclusion

- Equilibrium model confirms: both CRP and VRP predict the market risk premium.
- Contemporaneous regression approach leads to strong OOS predictability (compared to the traditional OOS).
- CRP works for longer horizons compared to VRP.
- Different return predictability horizons due to different horizons in risk channels predictions.

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