Strengthening International Courts and the Early Settlement of Disputes

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Technical Appendix

Definitions

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\begin{aligned}
\sigma(\hat{s}) &\equiv \{\pi \in [0,1] \mid s(\pi) = \hat{s}\} \\
g(\pi|\hat{s}) &\equiv \frac{f(\pi)}{\int\limits_{\sigma(\hat{s})} f(\hat{\pi})d\hat{\pi}} \text{ for all } \pi \in \sigma(\hat{s});\ 0 \text{ for all } \pi \notin \sigma(\hat{s}) \\
E[\pi|\hat{s}] &\equiv \int\limits_0^1 \pi g(\pi|\hat{s})d\pi \\
V &\equiv \frac{1}{v_D} + \frac{1}{v_P} \\
R_P(\pi) &\equiv \{(1-q)[pw\epsilon + (1-p)b] + q\pi a + q(1-\pi)c\}\, v_P - qk
\end{aligned}
$$

Equilibrium Selection and Characterization

Sketch of components of full proof of Proposition 2:

- If $k < k''$, there does not exist a universally divine pooling equilibrium. If $k'' \leq k$, then a universally divine pooling equilibrium exists. (Lemma 1)
- Note that two classes of strategy profiles are possible:
  1. Strategy profiles in which all demands are always rejected; i.e. $r(s(\pi))$ for all $\pi \in [0,1]$.
  2. Strategy profiles in which there exists at least one type of player who makes a demand that is accepted with positive probability.

We begin by restricting attention to the latter strategy profiles and prove properties of equilibria that include such strategy profiles. We refer to these as X-equilibria. (Definition 1) The former strategy profiles are ruled out on efficiency grounds in Lemmata 6 and 8.

- In any semi-separating X-equilibria, it must be the case that low types fully separate, high types demand $s = 1$, and $r(s = 1) < 1$. (Lemma 4)
- If $k \leq k'$ or $k'' \leq k$, then no universally divine semi-separating X-equilibrium exists. (Lemma 5)
- If $k \leq k'$, then the fully separating X-equilibrium is the unique efficient universally divine equilibrium. (Lemma 6)
- If $k' < k$, then no fully separating X-equilibrium exists. (Lemma 7)
- If $k \in (k', k'')$, then the semi-separating X-equilibrium is the unique efficient universally divine equilibrium. (Lemma 8)

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Lemma 1.

1. *If $k < k''$, then there does not exist a universally divine pooling equilibrium.*
2. *If $k'' \leq k$, then there exists a universally divine pooling equilibrium in which the plaintiff always demands $s = 1$ and this demand is always accepted.*

Proof of Lemma 1. In any possible pooling equilibrium, settlement demands take the form $s(\pi) = s^P$ for all $\pi \in [0, 1]$. Since D can't update her beliefs after observing s^P, D will accept a pooling demand s^P iff:

$$s^P \leq (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b]+qc+\frac{qk}{v_D}+q(a-c)E[\pi] \equiv \tilde{s}$$

1. If $k < k''$, then $\tilde{s} < 1$.
   - We begin by considering pooling demands for which $r(s^P) = 1$. Suppose $s^P \in (\tilde{s}, 1]$. Then $r(s^P) = 1$ and the plaintiff receives expected utility of $R_P(\pi)$. Consider type $\pi = 0$ and deviation $s' = (1-q)[pw\epsilon + (1-p)b] + qc - \delta$ for $\delta > 0$. This deviation offer would always be accepted by D, regardless of the form of off-the-equilibrium-path beliefs, since:

$$\begin{aligned}(1-s')v_D &= \{(1-q)[p(1-w\epsilon)+(1-p)(1-b)]+q(1-c)+\delta\}\, v_D \\ &> \{(1-q)[p(\epsilon-w\epsilon)+(1-p)(1-b)]+q\pi(1-a)+q(1-\pi)(1-c)\}\, v_D - qk\end{aligned}$$

for all $\pi \in [0,1]$. Such a deviation would be profitable for P of type $\pi = 0$ iff:

$$s' v_P > R_P(\pi = 0) \quad \Leftrightarrow \quad \delta < \frac{qk}{v_P}$$

Such a δ can always be found. So in any possible pooling equilibrium, it must be that $s^P \in [0, \tilde{s}]$.

- Now we consider pooling demands for which $r(s^P) = 0$; i.e. $s^P \in [0, \tilde{s}]$. Since $\tilde{s} < 1$, there always exists an interval of demands $(s^P, 1]$ that are not made in equilibrium. What does universal divinity imply about D's beliefs after observing such an out-of-equilibrium demand? Let $\hat{\rho}(\pi, s')$ denote the probability of rejection (i.e. going to trial) that makes a plaintiff of type π indifferent between the pooling offer s^P and a deviation s'. Then:

$$s^P v_P = \hat{\rho}(\pi, s') R_P(\pi) + [1 - \hat{\rho}(\pi, s')] s' v_P \;\Leftrightarrow\; \hat{\rho}(\pi, s') = \frac{s^P - s'}{\frac{R_P(\pi)}{v_P} - s'}$$

$$\Rightarrow \frac{\partial}{\partial \pi} \hat{\rho}(\pi, s') = \frac{(s' - s^P) q(a - c)}{\left(\frac{R_P(\pi)}{v_P} - s'\right)^2} \;>\; 0 \quad \text{for } s' > s^P$$

So universal divinity requires that conditional on receiving an off-the-equilibrium-path demand $s' \in (s^P, 1]$, D believes that P is of type $\pi = 1$. So D will accept the deviation offer $s' \in (s^P, 1]$ iff:

$$\begin{aligned} (1 - s') v_D \;&>\; \{(1-q)[p(\epsilon - \epsilon w) + (1-p)(1-b)] + q(1-a)\} v_D - qk \\ \Leftrightarrow s' \;&<\; (1-q)[p(1 - \epsilon + \epsilon w) + (1-p)b] + qa + \frac{qk}{v_D} \equiv \overline{s} \end{aligned}$$

Note that $s^P \leq \tilde{s} < \overline{s}$. So P of type $\pi = 1$ can always find a profitable deviation, which means that there does not exist a pooling equilibrium that satisfies the refinement of universal divinity.

2. If $k'' \leq k$, then $\tilde{s} \geq 1$ and $r(s^P) = 0$ for all pooling demands $s^P \in [0,1]$. So the universally divine pooling demand is $s^P = 1$. Since all types of P achieve their maximum possible payoff of v_P from this strategy profile, there is never incentive for any type to deviate, regardless of the form of off-the-equilibrium-path beliefs.

□

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Definition 1. *An* <u>*X-equilibrium*</u> *is a equilibrium with a strategy profile* (s, r) *such that there exists a type* $\pi \in [0,1]$ *for which* $r(s(\pi)) < 1$.

Lemma 2. *In any X-equilibrium, there exists a type $\hat{\pi} \in [0,1]$ s.t. $r(s(\pi)) < 1$ for all $\pi \in [0, \hat{\pi}]$.*

Proof of Lemma 2. This follows directly from the the monotonicity of $r(s(\pi))$ in π and Definition 1. □

Definition 2. *The X-space of an X-equilibrium is:*

$$\{\pi \in [0,1] \mid r(s(\pi)) < 1\}$$

Note that by Lemma 2, this set is convex and bounded below by $\pi = 0$.

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Lemma 3. *If there exists an interval of types, $[\underline{\pi}, \bar{\pi}]$, from the X-space that fully separate, it must be that:*

$$s(\pi) = (1-q)[p(1-\epsilon+w\epsilon) + (1-p)b] + \frac{qk}{v_D} + q\pi a + q(1-\pi)c \quad \textit{for all} \quad \pi \in [\underline{\pi}, \bar{\pi}] \tag{1}$$

Proof of Lemma 3. First, recall that $r(s(\pi))$ is increasing π. Consider $\pi, \pi' \in [\underline{\pi}, \bar{\pi}]$ such that $\pi < \pi'$. Monotonicity of $s(\pi)$ and full separation imply that $s(\pi) < s(\pi')$. Suppose $r(s(\pi)) = r(s(\pi'))$. Then type π has incentive to deviate to $s(\pi')$. So $r(s(\pi))$ is strictly increasing over $[\underline{\pi}, \bar{\pi}]$. This can only be true if D is playing a non-degenerate mixed strategy over this interval. So D must be indifferent over the choice of $r(s(\pi))$ over this interval. This is only true if eqn (1) holds. □

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Lemma 4. *In any universally divine semi-separating X-equilibrium, it must be the case that there exists a value $\hat{\pi} \in (0,1)$ such that: full separation occurs for $\pi < \hat{\pi}$; all types $\hat{\pi} < \pi$ demand $s = 1$; and $r(s = 1) < 1$.*

Proof of Lemma 4. We proceed through a series of sub-lemmata before making our final argument.

- *Claim 4.1: There does not ever exist a universally divine semi-separating X-equilibrium in which a pooling message $s^P < 1$ is sent by types in the X-space.*

Proof: Suppose that such an equilibrium exists. Choose an arbitrary pooling message sent by types in the X-space such that $s^P < 1$. Continuity of the demand-space (i.e. the unit interval) ensures that there will always exist a well-defined interval of feasible off-the-equilibrium path demands just above s^P. Consider deviation to such a demand s' from this interval, where $s' = s^P + \delta$ for small $\delta > 0$. Then universal divinity requires that after observing s', D believes her opponent is of type $\hat{\pi} \equiv max\ \sigma(s^P)$. So D will accept deviation s' iff:

$$\begin{gathered}(1-s')v_D \geq (1-q)[p(1-w)\epsilon + (1-p)(1-b)]v_D + q\hat{\pi}(1-a)v_D \\ + q(1-\hat{\pi})(1-c)v_D - qk \\ \Leftrightarrow qE[\pi|s^P](1-a) + q(1-E[\pi|s^P])(1-c) - \delta \geq q\hat{\pi}(1-a) + q(1-\hat{\pi})(1-c) \\ \Leftrightarrow q(a-c)(\hat{\pi} - E[\pi|s^P]) \geq \delta\end{gathered}$$

Such a δ can always be found. Type $\hat{\pi}$ will find it profitable to make such a deviation since individual rationality requires that $s^P v_P \geq R_P(\hat{\pi})$ and:

$$s'v_P = (s^P + \delta)v_P \geq r(s^P)r_P(\hat{\pi}) + [1 - r(s^P)]s^P v_P$$

- *Claim 4.2: There does not ever exist a universally divine X-equilibrium in which: full separation occurs in the X-space; and the X-space is a strict subset of the type-space; i.e.* $\{\pi \in [0,1] \mid r(s(\pi)) < 1\} \subset [0,1]$.

  Proof: Suppose that such an X-equilibrium exists. Then there exists a value $\hat{\pi} \in [0,1)$ s.t. types $\pi \in [0,\hat{\pi})$ fully separate and $r(s(\pi)) = 1$ for all $\pi > \hat{\pi}$.[1] Then by Lemma 3:

$$s(\pi) = (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b] + \frac{qk}{v_D} + q\pi a + q(1-\pi)c \text{ for all } \pi \in [0,\hat{\pi})$$

  Consider a type $\pi \in (\hat{\pi}, 1]$. He has no incentive to deviate and mimic the behavior of a type $\pi' \in [0,\hat{\pi})$ by demanding $s(\pi') \in [s(\pi = 0), s(\hat{\pi}))$ iff:

$$\begin{aligned} R_P(\pi) &\geq r(s(\pi'))R_P(\pi) + [1 - r(s(\pi'))]s(\pi')v_P \\ \Leftrightarrow R_P(\pi) &\geq s(\pi')v_P \\ \Leftrightarrow (\pi - \pi')q(a-c)v_P &\geq (1-q)p(1-\epsilon)v_P + qk\left(1 + \frac{v_P}{v_D}\right)\end{aligned}$$

  The term $(\pi - \pi')$ can be made arbitrarily small and the condition will fail. So there always exists a type $\pi \in (\hat{\pi}, 1]$ who has incentive to deviate to the demand made by a type $\pi' \in [0,\hat{\pi})$.

[1] The specific behavior at point $\hat{\pi}$ doesn't matter for this proof.

- Claims 4.1 and 4.2 imply that if a universally divine semi-separating equilibrium exists, all pooling demands are such that either (1.) $s^P = 1$, or (2.) $s^P < 1$ and $r(s^P) = 1$.
  - Suppose there exists a pooling message $s^P < 1$ such that $r(s^P) = 1$. Then the X-space is a strict subset of the type-space and by Claim 4.2 there is no full separation in the X-space. So there must exist another pooling demand $\hat{s}^P$ sent by types in the X-space, which implies that $r(\hat{s}^P) < 1$. By Claim 4.1, this is only possible if $\hat{s}^P = 1$. This violates monotonicity of $r(s)$.
  - Suppose $s^P = 1$ and $r(s^P = 1) = 1$. By the argument above, there must exist another pooling demand $\hat{s}^P$ such that $r(\hat{s}^P) < 1$. This is only possible if $\hat{s}^P = s^P = 1$, which contradicts the starting proposition that $r(s^P = 1) = 1$.

  So the only pooling demand is $s = 1$ and $r(s = 1) < 1$. Monotonicity of $s(\pi)$ ensures that full separation occurs for $\pi < \hat{\pi}$ and all types $\hat{\pi} < \pi$ demand $s = 1$.

□

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Lemma 5. *If $k \leq k'$ or $k'' \leq k$, then no universally divine semi-separating X-equilibrium exists.*

Proof of Lemma 5. Suppose that semi-separation occurs in a universally divine X-equilibrium. Then by Lemma 4 there exists an interval $[\hat{\pi}, 1]$ such that $s(\pi) = 1$ for all $\pi \in [\hat{\pi}, 1]$ where $\hat{\pi} \in (0, 1)$, and types below $\hat{\pi}$ separate.

- If $k \leq k'$, then $r(s = 1) = 1$, which contradicts Lemma 4.
- If $k'' \leq k$, then $r(s = 1) = 0$, so all types $\pi < \hat{\pi}$ have incentive to deviate from their separating demands to the pooling demand. This cannot constitute equilibrium behavior.

□

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Lemma 6. *If $k \leq k'$, then the universally divine equilibrium that maximizes efficiency is a fully separating equilibrium characterized by:*

$$\begin{aligned} s^*(\pi) &= (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b]+q\pi a+q(1-\pi)c+\frac{qk}{v_D} \\ & \quad \textit{for all } \pi \in [0,1] \\ r^*(s) &= \begin{cases} 0 & \textit{if } s < s^*(\pi=0) \\ 1-exp\left(-\frac{\Delta_\pi}{\Gamma}\right) & \textit{if } s \in [s^*(\pi=0), s^*(\pi=1)] \\ 1 & \textit{if } s > s^*(\pi=1) \end{cases} \end{aligned}$$

where $\Delta_\pi = s - s^*(\pi=0) = q\pi(a-c)$ *and* $\Gamma \equiv qkV + (1-q)p(1-\epsilon)$.

Proof of Lemma 6. By the previous results, only two classes of universally divine equilibria are possible for $k \leq k'$:

1. Fully separating equilibria in which $\{\pi \in [0,1] \mid r(s(\pi)) < 1\} = [0,1]$
2. Equilibria in which $r(s(\pi)) = 1$ for all $\pi \in [0,1]$.

- Characterization: For existence and characterization of the universally divine fully separating X-equilibria, see the Proof of Proposition 2 in the main Appendix for the paper.

- Efficiency: Consider an arbitrary equilibrium in which $r(s(\pi)) = 1$ for all $\pi \in [0,1]$. Joint welfare for the two players for a given value of π is:

$$U = R_P(\pi) + R_D(\pi)$$

  In contrast, joint welfare in the fully separating equilibrium is:

$$U = R_P(\pi) + R_D(\pi) + exp\left(-\frac{s^*(\pi)-\theta}{\Gamma}\right)\left[qk\left(1+\frac{v_P}{v_D}\right)+(1-q)p(1-\epsilon)v_P\right]$$

  So the fully separating equilibrium is more efficient than any possible equilibrium in which $r(s(\pi)) = 1$ for all $\pi \in [0,1]$.

  Now note that since D is playing a mixed strategy, she is indifferent over all choices of $\theta \leq s^*(\pi=0)$. In contrast, P's expected utility from an equilibrium strategy profile parameterized by θ for a value of π is:

$$\begin{aligned} EU_P(\theta|\pi) &= r^*(s,\theta)R_P(\pi) + [1-r^*(s,\theta)]s^*(\pi)v_P \\ &= R_P(\pi) + exp\left(-\frac{s-\theta}{\Gamma}\right)\left[qk\left(1+\frac{v_P}{v_D}\right)+(1-q)p(1-\epsilon)v_P\right] \end{aligned}$$

So P's expected utility for every value of π is increasing in θ. So $\theta = s^*(\pi = 0)$ maximizes efficiency.

□

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Lemma 7. *If $k' < k$, then there does not exist a universally divine fully separating X-equilibrium.*

Proof of Lemma 7. By Lemma 3, full separation requires:

$$s(\pi) = (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b]+q\pi a+q(1-\pi)c+\frac{qk}{v_D} \quad \text{for all } \pi \in [0,1]$$

This is well-defined iff:

$$s(\pi=1) \leq 1 \;\Leftrightarrow\; 0 \leq (1-q)[p(1-w)\epsilon+(1-p)(1-b)]v_D+q(1-a)v_D-qk \;\Leftrightarrow\; k \leq k'$$

□

Lemma 8. *If $k \in (k', k'')$, then the universally divine equilibrium that maximizes efficiency is a semi-separating equilibrium characterized by:*

$$s(\pi) = \begin{cases} (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b]+q\pi a+q(1-\pi)c+\frac{qk}{v_D} \\ \qquad \text{for all } \pi \in [0,\hat{\pi}) \\ 1 \quad \text{for all } \pi \in [\hat{\pi},1] \end{cases}$$

$$r(s) = \begin{cases} 0 & \text{if } s < s(0) \\ 1-exp\left(-\frac{\Delta_\pi}{\Gamma}\right) & \text{if } s \in [s(0),\hat{s}] \\ 1 & \text{if } s \in (\hat{s},1) \\ 1-exp\left(-\frac{\Delta_{\hat{\pi}}}{\Gamma}\right)\left[\frac{\hat{s}v_P-R_P(\hat{\pi})}{v_P-R_P(\hat{\pi})}\right] & \text{if } s = 1 \end{cases}$$

where $\hat{s} = (1-q)[p(1-\epsilon+w\epsilon)+(1-p)b]+q\hat{\pi}a+q(1-\hat{\pi})c+\frac{qk}{v_D}$ and $\Delta_\pi = s(\pi)-s(0) = q\pi(a-c)$.

Proof of Lemma 8. If $k \in (k', k'')$, then there do not exist universally divine pooling or fully separating X-equilibria. The only remaining possibilities are:

1. Semi-separating equilibria of the form outlined in Lemma 4.
2. Equilibria in which $r(s(\pi)) = 1$ for all $\pi \in [0,1]$.

- Characterization: For existence and characterization of the universally divine semi-separating X-equilibria, see the Proof of Proposition 2 in the main Appendix for the paper.

- Efficiency: Consider an arbitrary equilibrium in which $r(s(\pi)) = 1$ for all $\pi \in [0, 1]$. Joint welfare for the two players for a given value of π is:

$$U = R_P(\pi) + R_D(\pi)$$

In contrast, joint welfare in the semi-separating equilibrium above is:

$$\begin{aligned} U &= R_P(\pi) + R_D(\pi) + exp\left(-\frac{s^*(\pi) - \theta}{\Gamma}\right)[s^*(\pi)v_P - R_P(\pi)] \text{ for } \pi \in [0, \hat{\pi}) \\ &= R_P(\pi) + R_D(\pi) \\ &\quad + exp\left(-\frac{s^*(\pi) - \theta}{\Gamma}\right)\frac{\hat{s}v_P - R_P(\hat{\pi})}{v_P - R_P(\hat{\pi})}[s^*(\pi)v_P - R_P(\pi)] \text{ for } \pi \in [\hat{\pi}, 1] \end{aligned}$$

So the semi-separating equilibrium above is more efficient than any possible equilibrium in which $r(s(\pi)) = 1$ for all $\pi \in [0, 1]$.

Now note that since D is playing a mixed strategy, she is indifferent over all choices of $\theta \leq s^*(\pi = 0)$. In contrast P's expected utility from an equilibrium strategy profile parameterized by θ for a value of π is:

$$\begin{aligned} EU_P(\theta|\pi) &= r^*(s, \theta)R_P(\pi) + [1 - r^*(s, \theta)]s^*(\pi)v_P \\ &= R_P(\pi) + exp\left(-\frac{s^*(\pi) - \theta}{\Gamma}\right)[s^*(\pi)v_P - R_P(\pi)] \text{ for } \pi \in [0, \hat{\pi}) \\ &= R_P(\pi) \\ &\quad + exp\left(-\frac{s^*(\pi) - \theta}{\Gamma}\right)\frac{\hat{s}v_P - R_P(\hat{\pi})}{v_P - R_P(\hat{\pi})}[s^*(\pi)v_P - R_P(\pi)] \\ &\quad \text{for } \pi \in [\hat{\pi}, 1] \end{aligned}$$

So P's expected utility for every value of π is increasing in θ. So $\theta = s^*(\pi = 0)$ maximizes efficiency.

□

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The full proof of Proposition 2 follows from the existence and characterization results in the main Appendix of the paper and the conjunction of Lemmata 1 - 8 above.

Additional Comparative Statics

Proposition: For a low cost court or a dispute over a high value asset, as the plaintiff's share from either war (w) or negotiations after a case is dismissed (b) increases:

- settlement offers increase, and
- there is no effect on the probabilities that a settlement is rejected, trial takes place, or war occurs in equilibrium.

Proof of Proposition

$$\begin{aligned}
\frac{\partial s^*(\pi)}{\partial w} &= (1-q)p\epsilon > 0 \quad \text{and} \quad \frac{\partial s^*(\pi)}{\partial b} = (1-q)(1-p) > 0 \\
\frac{\partial r^*(s^*(\pi))}{\partial w} &= exp\left(-\frac{\Delta_\pi}{\Gamma}\right)\frac{\partial}{\partial w}\left[\frac{\Delta_\pi}{\Gamma}\right] = 0 \;\Rightarrow\; \frac{\partial T^*}{\partial w} = 0 \text{ and } \frac{\partial W^*}{\partial w} = 0 \\
\frac{\partial r^*(s^*(\pi))}{\partial b} &= exp\left(-\frac{\Delta_\pi}{\Gamma}\right)\frac{\partial}{\partial b}\left[\frac{\Delta_\pi}{\Gamma}\right] = 0 \;\Rightarrow\; \frac{\partial T^*}{\partial b} = 0 \text{ and } \frac{\partial W^*}{\partial b} = 0
\end{aligned}$$