

Earning to Give:

Occupational Choice for Effective Altruists*

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Abstract

Effective altruists wish to do good while optimizing the social performance they deliver. We apply this principle to the labor market. We determine the optimal occupational choice of a socially motivated worker who has two mutually exclusive options: a job with a for-profit firm and a lower-paid job with a nonprofit. We construct a model in which a worker motivated only by pure altruism will work at a relatively high wage for the for-profit firm and then make charitable contributions to the nonprofit; this represents the “earning to give” option. By contrast, the occupational choice of a worker sensitive to warm glow (“impure altruism”) depends on her income level. While the presence of “warm glow” feelings would seem to clearly benefit charitable organizations, we show that impure altruism can create distortions in labor market choices. In some cases, warm glow feelings may push the worker to take a job with the nonprofit, even when it is not optimal for the nonprofit.

1. Introduction

The effective altruism movement (Singer, 2015) combines the philanthropic desire to do good with the economic principle of cost-effectiveness. Effective altruists prize a hard-headed approach to maximizing impact on people affected by charities, in contrast to giving motivated by the warm glow of feeling that one has done good. This paper brings this principle to the labor market, by considering a worker with pro-social motivations who must choose between a regular job in a for-profit firm and a lower-paying job in a nonprofit. Working for the for-profit allows a larger financial donation, while working directly for the non-profit permits contributions of uncompensated effort. In this way, we build a theoretical model translating effective altruists' imperative of combining giving time and money in a way that maximizes their contribution to society while ensuring decent living conditions for themselves.

Charitable giving is increasingly attracting scholarly attention (Evren & Minardi, 2017; Ottoni-Wilhelm et al., 2017; Karlan & Woodson, 2017; Kolm, 2014). Economists recognize two different motives for giving: pure altruism and warm-glow feelings (also known as “impure altruism”, Andreoni 1989). In pure altruism, the donor gets utility from a public good, represented as the output of a charitable nonprofit in our model. In contrast, warm glow is typically associated with the donor's positive feelings about their own actions, as a direct benefit coming from the act of giving. Both types of altruism apply to money and cash donations alike.

The presence of “warm glow” feelings would seem to clearly benefit charitable organizations, but, from the standpoint of the recipient nonprofit, things can look different. The capacity to give is shaped in part by outcomes in the labor market, and impure altruism can create distortions in labor market choices.¹ Workers motivated by mission are valuable but working directly for nonprofits may not be the most effective way that workers can help. By working for a for-profit firm at a high wage instead, workers may be able to donate a sum to the nonprofit that exceeds the value of the time that can be transferred through labor donation (Mathews 2013). In this way, we show a labor market analogue to findings of

¹ People help others by sharing money or time. A 2013 Gallup poll, for example, found that 83 percent of Americans reported giving money to charity and 65 percent volunteered in the prior year (Gallup, 2013). *The World Giving Index 2014* (Charities Aid Foundation, 2014), shows that the United States is the leader in charitable giving, possibly because in other countries the state provides services that are provided by the private sector in the United States. Other high-donating countries in the index include Myanmar, Ireland, Malta, Canada, the Netherlands, United Kingdom, and Iceland. The index counts the number of people giving, not the size of contributions.

inefficiencies due to warm glow in experiments on charitable giving (Null 2011, Karlan & Woodson 2017). Our model formalizes this key dichotomy by parametrizing separately the degrees of pure and impure altruism of the motivated worker.

In the standard theory of labor supply, workers first select jobs that offer the highest wages and then allocate their earnings to consumption and other spending, including pro-social ends such as supporting relatives, communities, or charities. In practice, workers face a broader set of choices. Rather than first maximizing income, they can also consider positions that permit helping others directly through their work. In 2012, for example, charitable organizations accounted for 10 percent of all private sector employment in the United States (BLS 2014).² Those positions, though, may pay less than other jobs. The 2007 National Compensation Survey of the US Bureau of Labor Statistics, for example, shows that non-profit workers earned an average of \$34.24 per hour versus \$41.86 per hour for workers working for for-profit firms—a wage gap of 18 percent (Butler 2009).

Why do workers choose positions at nonprofits despite lower pay? One reason for the wage gap is non-monetary job attributes that create compensating differentials: nonprofits often provide warm glow from involvement in the charitable enterprise (Benz 2005, Brown *et al.* 2013). The utility of those attributes at least partially compensates for higher wages available elsewhere. A second reason for the wage differential is labor donation: workers seeking to support the mission of the nonprofit may accept a lower wage as a way to transfer resources to the nonprofit (Preston 1989, Shahrier & Kotani 2014). These workers may not feel a warm glow or even particularly like the job, but they support the nonprofit’s mission by accepting a lower wage or working harder without extra compensation.³

Our simple model builds from these empirical patterns. Our results show that the motivational balance between pure and impure altruism is key in the decision of earning to give. If she has resources sufficient to cover the basic costs of living, the pure altruist earns to give, and the resulting equilibrium is optimal for the nonprofit. The presence of impure

² Nonprofit jobs are, of course, not the only jobs that allow workers to use their time toward social goals, nor are all nonprofit jobs particularly special with regard to social contributions. Leete (2001) and Ruhm and Borkoski (2003), for example, show evidence that most nonprofit jobs in the US do not pay lower wages than for-profit jobs, especially once worker attributes are considered. They find wage gaps for some sub-sectors and occupations, though. The Bureau of Labor Statistics shows particularly large wage gaps for professionals and managers (Butler, 2009).

³ An additional reason (that we do not pursue here) is unobserved productivity. Workers in nonprofits may be intrinsically less productive due to unobserved qualities or weaker monitoring technologies, or employers in the private sector may be more willing to pay efficiency wages. Both features would drive a wedge between wages in the two sectors.

altruism reshuffles the deck. The motivated worker then arbitrates between pure and impure altruism, and her occupational choice depends on the wage gap between working at a for-profit versus a nonprofit and on the worker's non-labor income. The high-income worker with pure and impure altruism systematically decides against the earning-to give option as her budget constraint allows her to capture the warm glow effect of the compassionate occupation while compensating—at least partially—the nonprofit with a grant. By contrast, a tight cash constraint can push the worker with mixed altruism to pick the earning-to-give option because she would be unable to compensate the lack of labor donation by extra cash donation. Yet, the higher her impure altruism relative to pure altruism, the less likely this choice. The model shows why giving time and giving money are not consistently substitutes or complements, and we show why low-income workers are more likely than others to donate time rather than money.

2. Model Setup

We build from the literature on donations and volunteer labor (e.g., Brown & Lankford, 1992; Andreoni *et al.*, 1996; Brown *et al.* 2013; Bauer *et al.* 2013), which investigates the complementarity of donating money and volunteering time. Volunteering is a particular form of work, and in this paper, in contrast, we focus on full-time paid work in professional roles. We incorporate charitable giving by starting with an altruist who wants to use her time to help others. She could seek a for-profit job that pays well and allows her to make substantial charitable donations to a nonprofit, or she could work for the nonprofit directly. She is more productive than other workers at the nonprofit, so her labor donation is relatively valuable, but it comes with opportunity costs.

We build a simple static model explaining the occupational choice of workers motivated by altruism. Unlike other workers, these “motivated” workers are employees who are intrinsically motivated by the chance to contribute to an organization’s mission. Along these lines, Akerlof and Kranton (2000) develop a theory of identity that provides foundations for worker motivation. Besley and Ghatak (2005) explore how the presence of motivated workers can relax principal-agent problems. Ashraf *et al.* (2014) use a randomized trial to show how the hiring of motivated workers affects productivity more strongly than wage incentives (in their case, career motivations dominate social motivations).

The question here is why some altruistically-motivated workers prefer to work directly for nonprofits while others prefer to work in for-profit firms and make cash donations to nonprofits. To pick an extreme example, why do Bill and Melinda Gates choose to run their foundation themselves, while Warren Buffet focuses on investing and gives away billions?

Motivated workers are joined in the labor market by “regular” workers. Regular workers are defined here in the specific sense that they do not derive utility from the output of the non-profit, although they may benefit from “warm glow” feelings from simply working for the nonprofit. They are as productive and hard-working as “motivated” workers when working in the private sector.

Firms. There are a given number of firms active in the same competitive market: one is a nonprofit (NP) firm, and the others are for-profit (FP) firms. The FP firms are standard profit-maximizing firms. In contrast, the nonprofit distributes its output for free to those in need. It is able to produce thanks to an exogenous subsidy S , and endogenous labor and cash donations. Its production function is:

$$Q = f(K), \quad f' > 0. \tag{1}$$

where Q is output and K is capital.

The nonprofit hires a single worker. The for-profit firm pays the market wage w_{FP} to their workers, while the nonprofit pays wage w_{NP} , where $w_{NP} \leq w_{FP}$. Why would a regular (non-motivated) worker ever take a non-profit job? Although regular workers do not gain utility directly from the non-profit’s output, they are assumed to enjoy non-pecuniary benefits from non-profit employment. This critical distinction simplifies the model and allows us to focus on the decisions of the motivated workers (Özgür & Minardi, 2017). The benefits received by regular workers provide a compensating wage differential, and wages are set so that regular workers are indifferent between employment in the two sectors. We call this a “warm glow” effect, but it could stem from other non-pecuniary benefits.

The compensating wage differential is such that the wage gap between the for-profit and non-profit sector is exactly offset by the non-monetary compensation associated with warm glow. Therefore, w_{NP} can be considered as the market wage for the worker working with the nonprofit. We denote by Δ the wage gap due to warm-glow:

$$\Delta = w_{FP} - w_{NP} \geq 0$$

Workers. There are two types of workers in the market. As above, regular workers are indifferent between the two types of firms since the warm glow effect counteracts the wage gap. In contrast, the motivated worker *really* cares about nonprofit's output. Her utility U depends on both personal consumption C and the nonprofit's output Q . To work with explicit expressions, we assume that the motivated worker has a separable logarithmic utility function which is the log-equivalent to Cobb-Douglas utility:

$$U_M(C, Q, I_{NP}) = U(C, Q) + c I_{NP} = \ln C + b \ln Q + c I_{NP}, b > 0, c \geq 0 \quad (2)$$

where I_{NP} is a binary variable taking value one when the motivated worker works with the NP:

$$I_{NP} = \begin{cases} 1 & \text{if NP firm} \\ 0 & \text{if FP firm} \end{cases}$$

The coefficient b captures the degree of motivation (or pure altruism) of the motivated worker, while coefficient c represents their sensitivity to warm glow (or impure altruism). The altruist's utility has two components with different origins. Utility gains through pure altruism come from the output of the nonprofit firm while the warm glow component is directly associated with the binary variable I_{NP} capturing the choice of "compassionate career" (using the language of Pryor and Mitchell 2015).

Since we assume no special link between Δ and c , the motivated worker would not necessarily respond to warm glow to the same extent as regular workers. An effective altruist may actively put aside their warm glow feelings in making choices, for example. Since the motivated worker especially cares about the output of the nonprofit, she could be willing to work with the for-profit firm instead, if it means that she could earn substantially more money to donate to the nonprofit. This is the idea underpinning the tenets of the "earning to give" principle, which as Muyskens (2017) puts it, is the ascetic "other half of effective altruism" that complements the search for cost-effective charities.

At the same time, a motivated worker working with the nonprofit demands wage $\widetilde{w}_{NP} \leq w_{NP}$. The motivation combines warm glow and labor donation. Her labor donation is the wage sacrifice made with respect to working with the for-profit firm. This sacrifice can take the form of either partial volunteering, or increased productivity without compensation. To simplify the presentation, we represent the labor donation V in terms of the wage differential with respect to the for-profit wage w_{FP} :

$$V = w_{FP} - \widetilde{w}_{NP}.$$

By assumption, we have:

$$V \geq \Delta = w_{FP} - w_{NP}. \quad (3)$$

The motivated worker's total income is the sum of her wage (w_{FP} or \widetilde{w}_{NP} , depending of her employer) and her non-labor income y (Menchik & Weisbrod, 1987). She has two—combinable—ways of expressing her motivation toward the non-profit: by giving time (labor donation) and by giving money (cash donation).

Labor and Cash Donations. The nonprofit's production function is assumed to be:

$$f(K) = K^a \quad a \geq 0. \quad (4)$$

The motivated worker has the opportunity to make a cash donation to the nonprofit regardless of which firm she's working with. As a result, the nonprofit firm collects a donation, say D , from the motivated worker. If the motivated worker instead selects a job with the for-profit firm and thus earns a higher wage, she can make an even larger donation to the nonprofit.

The Nonprofit Firm's Problem. According to its social mission, the nonprofit firm maximizes output $Q = K^a$ under a budget constraint that depends positively on exogenous subsidy and the cash donation from the motivated worker and negatively on wages:

$$K = \begin{cases} S - \widetilde{w}_{NP} + D & \text{if altruistic worker} \\ S - w_{NP} + D & \text{if regular worker} \end{cases} \quad (5)$$

Note that the budget does not directly depend on the type of worker hired by the nonprofit. However, indirectly it does since donation D depends on the wage—and thus the employer—of the motivated worker. From Eq. (4), the objective function of the nonprofit can be written:

$$Q = \begin{cases} (S - \widetilde{w}_{NP} + D)^a & \text{if altruistic worker} \\ (S - w_{NP} + D)^a & \text{if regular worker} \end{cases} \quad (6)$$

In either case, the nonprofit pays a lower wage than its for-profit counterpart, either because of warm glow (if a regular worker) or because of labor donation (if a motivated worker). We assume that:

$$S > w_{NP} (\geq \widetilde{w}_{NP}). \quad (7)$$

Under this technical condition, the nonprofit can afford to pay its worker's wage regardless of the size of the cash donation.

The Motivated Worker's Problem. The motivated worker makes three joint decisions: where to work (labor donation), how much to give (cash donation), and how much to consume. The choices are formalized as follows: 1) the binary job choice between the for-profit firm or the nonprofit, 2) the amount D donated to the nonprofit, and 3) the consumption level, C . The motivated worker maximizes utility $U_M(C, Q, I_{NP})$ under the budget constraint:⁴

$$y + w - D = C \quad (8)$$

where

$$w = \begin{cases} w_{FP} & \text{if her employer is the for-profit firm} \\ \overline{w}_{NP} & \text{if her employer is the nonprofit} \end{cases} \quad (9)$$

We solve this problem in two steps. First, we derive the optimal values of D and C for the nonprofit workers employed by the FP firm and the nonprofit, respectively. Then, we compare the two optimal utilities to determine the NP worker's choice of employer.

Beyond maximizing profit (for-profit firm) or output (nonprofit), we assume that the two firms are passive. Since the regular workers are indifferent between the two types of firms, ultimately the decisive job choices are made by motivated workers. Henceforth, the situation where the motivated worker opts for the nonprofit is referred to as the *compassionate-career case*, whereas the other combination where the motivated worker goes with a for-profit firm is called the *earning-to-give case*.

⁴ If donations are tax-deductible (rate θ) the constraint becomes: $y + w - (1 - \theta)D = C$.

3. Optimal Donations in the Compassionate-Career and Earning-to-Give Cases

3.1 The Compassionate-Career Case

Let us first consider the case of the motivated worker finding employment with a charitable organization. In the model here, the motivated worker's utility is linked to the nonprofit's productivity, which in turn depends on the type of worker that this firm has hired. The motivated worker knows that if she opts to work for the for-profit firm, the nonprofit will end up with a regular worker.

When the motivated worker takes the job with the nonprofit, the regular worker is irrelevant to our problem, and we have:

$$Q = (S - \widetilde{w}_{NP} + D)^a. \quad (10)$$

The motivated worker's problem is then:

$$\text{Max } U(C, Q) + c \quad \text{s.t. } C = y + \widetilde{w}_{NP} - D \quad (11)$$

To some extent, this model mimics the standard two-good consumption model, where output Q acts as a second consumption good (besides actual consumption C). However, an important difference comes from the fact that part of the total budget—namely wage \widetilde{w}_{NP} —has a direct and negative impact on the quantity of the “second good.” For the rest, the problem is standard, and optimization will determine the optimal quantity consumed of the “second good” via optimal donation D .

Ex ante, the total impact of the non-profit wage \widetilde{w}_{NP} on utility is ambiguous because there is a trade-off. On one hand, a higher \widetilde{w}_{NP} increases the budget of the motivated worker; on the other, it reduces the budget of the nonprofit. The optimization will establish which impact dominates.

The motivated worker maximizes:

$$U(D) = \ln[y + \widetilde{w}_{NP} - D] + b a \ln[S - \widetilde{w}_{NP} + D] + c \quad (12)$$

Let us assume for the moment that the non-negativity conditions are fulfilled. The first order condition is:⁵

$$-(S - \widetilde{w}_{NP} + D) + ba[y + \widetilde{w}_{NP} - D] = 0. \quad (13)$$

It follows that:

$$D^* = \widetilde{w}_{NP} + \frac{-S+bay}{1+ba} = w_{FP} - V + \frac{-S+bay}{1+ba} \quad (14)$$

The optimal cash donation depends positively on wage \widetilde{w}_{NP} . However, we have:

$$\frac{\partial D^*}{\partial V} = -1, \quad (15)$$

which means that any extra dollar provided as a labor donation reduces the cash donation by the same amount. Labor donation and money donation interact as communicating vessels.

The nonprofit's optimal output is:

$$Q^* = (S - \widetilde{w}_{NP} + D^*)^a = \left(\frac{ba(y+S)}{1+ba}\right)^a \quad (16)$$

This implies that:

$$\frac{\partial Q^*}{\partial V} = 0. \quad (17)$$

This optimal output does not depend on the wage the nonprofit is paying to the motivated worker. This is the direct consequence of Eq. (15) stating that any increase in labor donation translates into a same-size decrease in cash donation.

Last, the motivated worker's optimal consumption is:

$$C^* = \frac{S+y}{1+ba} \quad (18)$$

and:

$$\frac{\partial C^*}{\partial V} = 0. \quad (19)$$

⁵ This result does not hold for $b = 0$ (regular worker) because then we then have a corner solution $D^* = 0$ (no interior solution since $\ln C$ is decreasing with respect to D)

Not surprisingly, the labor donation has no impact on the consumption of the motivated worker in the compassionate-career situation. The optimal cash donation makes both Q^* and C^* insensitive to labor donation. Hence, the wage of the motivated worker employed by the nonprofit does not affect her utility.⁶ Provided that her non-labor income is sufficient to produce an interior solution to utility maximization, the motivated worker might as well accept a zero wage.⁷ This is clearly unrealistic as a general proposition but applies in some extreme cases (e.g., to Bill and Melinda Gates).

As expected, cash donations increase with y , the worker's non-labor income, and we have:

$$0 < \frac{\partial D^*}{\partial y} = \frac{ba}{1+ba} < 1 \quad (20)$$

The motivated worker shares her non-labor income with the nonprofit. She donates proportion $\frac{ba}{1+ba}$ for production and keeps the remaining fraction $\frac{1}{1+ba}$ for consumption. The donated fraction increases with both the motivated worker's degree altruism (b) and her productivity (a).

Interestingly, optimal donation D^* depends negatively on S , the external subsidy received by the nonprofit:

$$-1 < \frac{\partial D^*}{\partial S} = -\frac{1}{1+ba} < 0. \quad (21)$$

As a consequence, the motivated worker's consumption depends positively on this subsidy. When the nonprofit has more external subsidy, the motivated worker does not need to donate as much money to ensure that the nonprofit can generate a given level of output. Existing subsidies thus allow motivated workers to grant less money to charities, and so increase their own consumption. The subsidy provider produces a positive externality on the utility of the individuals who share the social concern of the nonprofit. Interestingly, Q^* , the nonprofit's output at the motivated worker's optimum, is proportional to C^* , the optimal consumption of this worker. This perfectly aligns the motivated worker's two objectives. The coefficient of proportionality is $b > 0$, the worker's degree of altruism.

⁶ The utility function here is quite standard and simplifies results. A more general specification would deliver similar trade-offs, but with more interactions.

⁷ We exclude tax deductions here. If cash donations were deductible, the tax deduction would work as a "utility machine" since the altruistic worker could turn tax benefits into output and consumption.

Until now, we have disregarded the non-negativity restrictions. In sum, when non-negativity restrictions do not bind, we have:

$$D^* = \widetilde{w}_{NP} + \frac{-S+bay}{1+ba} \quad (22a)$$

$$C^* = \frac{S+y}{1+ba} \quad (22b)$$

$$Q^* = \left(\frac{ba(y+S)}{1+ba} \right)^a = (ba C^*)^a \quad (22c)$$

Equation (22a) shows that the exogenous subsidy S has a negative impact on the optimal cash donation D^* . This impact is reminiscent of the crowding-out effect implied by pure altruism. When donors are motivated by the nonprofit's output only, their incentive to donate decreases with donations made by others (Warr, 1982; Ottoni-Wilhelm et al., 2017).

We now add the non-negativity restrictions to the picture. In fact, Eqs. (22b) and (22c) show that C^* and Q^* are always non-negative. But from Eq. (22a) it appears that for some parameter configurations, D^* can be negative. Intuitively, these configurations happen when the motivated worker is not wealthy enough to insure her basic consumption needs *and* make a cash donation (on top of labor donation). In other words, when the non-negativity constraint binds, the optimal cash donation is zero and the motivated worker's consumption depends on her wage. Such a "labor-only donation" typically arises when the motivated worker has low non-labor income y , and/or when the nonprofit wage is low, and/or when the nonprofit is highly subsidized. The impact of a high subsidy is exacerbated by the productivity of the motivated worker (her labor donation) and mitigated by her level of altruism (her willingness to donate).

Taking account of the non-negativity restrictions yields the following Proposition for the compassionate-career situation.

Proposition 1: The optimal strategy for the motivated worker in the compassionate-career situation is given by:

$$D_{CC}^* = \text{Max} \{0, D^*\}$$

$$C_{CC}^* = \begin{cases} \frac{S+y}{1+ba} & \text{if } D^* > 0 \\ y + \widetilde{w}_{NP} & \text{if } D^* = 0 \end{cases}$$

$$Q_{CC}^* = \begin{cases} \left(\frac{ba(y+S)}{1+ba}\right)^a = (ba C_{CC}^*)^a & \text{if } D^* > 0 \\ (S - \overline{w}_{NP})^a & \text{if } D^* = 0 \end{cases},$$

$$\text{where } D^* = \overline{w}_{NP} + \frac{-S+bay}{1+ba}.$$

To ease interpretation, we label the two cases as the “modest income” $\left(\overline{w}_{NP} + \frac{-S+bay}{1+ba} \leq 0\right)$ and the “high income” $\left(\overline{w}_{NP} + \frac{-S+bay}{1+ba} > 0\right)$ compassionate-career cases, respectively. These two categories can be defined with respect to the level of for-profit market wage $w_{FP} = \overline{w}_{NP} + V$.

Definition 1: A motivated worker has a modest (resp. high) income if: $w_{FP} <$ (resp. \geq) $\frac{-S+ba y-(1+ba)V}{1+ba}$.

The right-hand-side threshold in Definition 1 is specific to the motivated worker. It takes into account not only her personal non-labor income (y) but also the subsidization of the nonprofit (S). This threshold is higher when the nonprofit is better subsidized.

In the compassionate-career situation, the modest-income worker ends up donating labor only, meaning that she works with the nonprofit for a below-market wage. In contrast, the high-income worker gives both money and labor to the nonprofit. The combination of these two gifts makes her insensitive to the actual wage. Meanwhile, her consumption starts depending (positively) on the subsidy received by the nonprofit.

3.2 The Earning-to-Give Case

Warren Buffet has given billions of dollars to charity, but he has kept his focus on investing through Berkshire Hathaway, his for-profit company. He’s fortunate that Bill and Melinda Gates (and other motivated people) run the charities which he supports. We consider the case in which the nonprofits would instead have to hire a non-motivated worker.

When the nonprofit hires a regular worker, the nonprofit’s production function is given by:

$$Q = (S - w_{NP} + D)^a \tag{23}$$

Meanwhile, the motivated worker gets her wage from the for-profit firm, and her problem becomes:

$$\text{Max } U(C, Q) \quad \text{s.t. } C = y + w_{FP} - D \quad (24)$$

or equivalently:

$$U(D) = \ln[y + w_{FP} - D] + ba \ln[S - w_{NP} + D] \quad (25)$$

Her first order condition is:

$$-(S - w_{NP} + D) + b a[y + w_{FP} - D] = 0$$

Ignoring the non-negativity conditions, we obtain:

$$D^* = \frac{-(S - w_{NP}) + ba(y + w_{FP})}{1 + ba}$$

Substituting for the for-profit wage $w_{FP} = w_{NP} + \Delta$, we have:

$$D^* = w_{NP} + \frac{-S + ba(y + \Delta)}{1 + ba}$$

The wage gap due to warm glow Δ acts as an extra amount of non-labor income. In sum, when non-negativity restrictions do not bind, the optimal donation, output and consumption are given by:

$$D^* = w_{NP} + \frac{-S + ba(y + \Delta)}{1 + ba} \quad (26a)$$

$$C^* = \frac{S + y + \Delta}{1 + ba} \quad (26b)$$

$$Q^* = \left(\frac{ba(y + S + \Delta)}{1 + ba} \right)^a = (ba C^*)^a \quad (26c)$$

Again, the nonprofit wage is irrelevant to the motivated worker's consumption and to the nonprofit's output. However, the wage gap matters. Indeed, the extra money brought by working with the for-profit firm gives an opportunity for granting a higher donation to the nonprofit.⁸ Like for D^* in Equation (22a), we observe a crowding-out effect due to the negative impact of external subsidy S on the worker's donation D^* . In addition, the derivative is the same in both cases $\left(\frac{-1}{1 + ba} \right)$, which shows that the intensity of the crowding-out effect observed for money donations is not affected by the occurrence of labor donation.

⁸ If cash donations are deductible, this effect is even stronger.

Like in the compassionate-career situation, the motivated worker is better off giving money to a highly subsidized non-profit. (This could explain why Warren Buffet finances the Gates Foundation rather than a smaller charity. His marginal utility of donation is higher when he donates to a charity largely funded by others.)

When the non-negativity restrictions are taken into account, we obtain the following proposition.

Proposition 2: The optimal strategy for the motivated worker in the earning-to-give situation is given by:

$$D_{ETG}^* = \text{Max} \{0, D^*\}$$

$$C_{ETG}^* = \begin{cases} \frac{S+y+\Delta}{1+ba} & \text{if } D^* > 0 \\ y + w_{NP} + \Delta & \text{if } D^* = 0 \end{cases}$$

$$Q_{ETG}^* = \begin{cases} \left(\frac{ba(y+S+\Delta)}{1+ba}\right)^a & \text{if } D^* > 0 \\ (S - w_{NP})^a & \text{if } D^* = 0 \end{cases},$$

$$\text{where } D^* = w_{NP} + \frac{-S+ba(y+\Delta)}{1+ba}.$$

To interpret these results, keep in mind that in the earning-to-give situation there is no labor donation. Thus, if the motivated worker fails to earn sufficient income to make a cash donation, she gives nothing to the nonprofit even though she is motivated and gains utility from its output. Intuitively, this no-donation situation is less prevalent than in the compassionate-career case since the motivated worker gets a higher wage with the FP firm. Likewise, one expects that, all other things equal, the cash donation is higher in the earning-to-give situation than in the compassionate-career situation. The next proposition formalizes this idea.

Proposition 3: The optimal donation in the earning-to-give case is not smaller than in the compassionate-career case: $D_{ETG}^* \geq D_{CC}^*$.

Proof: see Appendix A.

According to Proposition 3, among the previously denominated modest-income workers $\left(\overline{w_{NP}} + \frac{-S+bay}{1+ba} \leq 0\right)$, some are wealthy enough $\left(\overline{w_{NP}} + \frac{-S+bay}{1+ba} \leq 0 < w_{NP} + \frac{-S+ba(y+\Delta)}{1+ba}\right)$ to work with the for-profit firm and make cash donations to the nonprofit, while others

$\left(w_{NP} + \frac{-S+ba(y+\Delta)}{1+ba} \leq 0\right)$ end up making no cash donation whatever their occupation. To acknowledge this new distinction, we break down the former “modest income” category into “low income” and middle income.” The high-income category remains unchanged, however the workers in this category are wealthier in the earning-to-give situation (i.e., no labor donation), and are thus able to make larger cash donations.

Definition 2: A modest-income motivated worker has a low (resp. middle) income if: $w_{FP} \leq$ (resp. $>$) $\frac{-S+ba y-\Delta}{1+ba}$.

Table 1 summarizes the options offered to motivated workers with low, middle, and high income, respectively. The table shows that the only feasible donation for low-income workers is labor donation, through higher productivity for a lower wage. The pattern was highlighted by Shahrier and Kotani (2014) in studying the ways that citizens contributed to post-cyclone relief in Bangladesh. These workers cannot afford giving money away even though they are motivated. Whether the labor-donation scenario is optimal or not depends on the parameters of the model, and notably on the degree of altruism b .

The middle-income workers can choose between only-labor and only-money donations. They cannot combine both types of donations. The extra wage obtained from working with the for-profit firm is needed for making them able to donate money to the nonprofit. Again, the optimal occupation rests on the values of the parameters at stake. Note, however, that the utility obtained in the money-donation case is independent of the nonprofit wage. Still, this result only holds for wage levels that fulfill the inequality that make total income high enough to belong to the middle-income category. In fact, the width of the middle-income category heavily rests on the wage gap Δ , i.e. the non-monetary compensation in the form of warm glow.

Last, the high-income worker will always give money to the nonprofit. But she has two options regarding labor donation: either the Bill and Melinda Gates option (compassionate-career case) or the Warren Buffet option (earning-to-give case). The next section discusses how the choice is made between these two options.

Table 1: Summary of the Possible Outcomes

Worker Situation	Low income $W_{FP} \leq \frac{-S+ba y-\Delta}{1+ba}$	Middle income $\frac{-S+ba y-\Delta}{1+ba} < W_{FP} \leq \frac{-S+ba y-(1+ba)V}{1+ba}$	High income $W_{FP} \geq \frac{-S+ba y-(1+ba)V}{1+ba}$
Compassionate Career	Labor donation No cash donation	Labor donation No cash donation	Labor donation Cash donation
Earning-to- give	No labor donation No cash donation ⁹	No labor donation Cash donation	No labor donation (Higher) cash donation

4. Motivated Worker's Occupational Choice

Ultimately, the motivated worker is confronted with the choice of working with the nonprofit (compassionate-career case) and getting utility U_{CC} , or working with the for-profit (earning-to-give case) and getting utility U_{ETG} , where:

$$U_{CC} = \ln C_{CC}^* + b \ln Q_{CC}^* + c \quad (30)$$

$$U_{ETG} = \ln C_{ETG}^* + b \ln Q_{ETG}^* \quad (31)$$

The difference between the utilities in (30) and (31) may be split in three terms, the first related to optimal consumption, the second comparing optimal output of the nonprofit, and the last referring to the personal preference for working with the nonprofit.

$$\Delta U = U_{CC} - U_{ETG} = \Delta \ln C^* + b \Delta \ln Q^* + c \quad (32)$$

where:

$$\Delta \ln C^* = \ln C_{CC}^* - \ln C_{ETG}^* \quad (33)$$

⁹ Note that even if the altruistic worker makes no donation at all to the nonprofit, she still gains utility from the subsidized output of the nonprofit.

$$\Delta \ln Q^* = \ln Q_{CC}^* - \ln Q_{ETG}^* \quad (34)$$

To explore the signs of $\Delta \ln C^*$ and $\Delta \ln Q^*$, the next propositions compare optimal consumption and output in the compassionate-career and earning-to-give cases, respectively.

Proposition 4: Optimal consumption in the earning-to-give case is not smaller than in the compassionate-career case: $\Delta \ln C^* = \ln C_{CC}^* - \ln C_{ETG}^* \leq 0$.

Proof: see Appendix B

Proposition 5: Optimal nonprofit output in the earning-to-give case is not smaller than in the compassionate-career case: $\Delta \ln Q^* = \ln Q_{CC}^* - \ln Q_{ETG}^* \leq 0$.

Proof: see Appendix C

Together Propositions 4 and 5 yield the next theorem describing the final decision of the motivated worker.

Theorem 1

- (i) If $c = 0$ (no impure altruism), the motivated worker always opts for the earning-to-give solution;¹⁰
- (ii) If $c > 0$ (impure altruism), the motivated worker arbitrates between pure and impure altruism as follows:
 - She opts for the compassionate-career solution if $c > \bar{c}$
 - She opts for the earning-to-give solution if $c < \bar{c}$
 - She is indifferent if $c = \bar{c}$

where threshold \bar{c} varies according to the income situation:

$$\bar{c} = \begin{cases} \ln \left[\frac{y+w_{FP}}{y+w_{FP}-V} \left(\frac{S-w_{FP}-\Delta}{S-w_{FP}-V} \right)^{ab} \right] & \text{for low income} \\ \ln \left\{ ab^{ab} \left[\frac{S+y+\Delta}{(1+ba)(y+w_{FP}-V)} \right]^{ab+1} \right\} & \text{for middle income} \\ \ln \left[\left(\frac{S+y+\Delta}{S+y} \right)^{ab+1} \right] & \text{for high income} \end{cases}$$

¹⁰ Yet, as Table 1 indicates, this situation could ultimately correspond to zero donation from workers in need, i.e. when $w_{FP} \leq \frac{-S+ba y-\Delta}{1+ba}$.

Proof: see Appendix D

A few special cases of Theorem 1 stand out. First, if there is no warm glow for the regular workers: ($\Delta = 0$) but there is possible labor donation ($V > 0$), then:

$$\bar{c} = \begin{cases} \ln \left[\frac{y+w_{FP}}{y+w_{FP}-V} \left(\frac{S-w_{FP}}{S-w_{FP}-V} \right)^{ab} \right] > 0 & \text{for low income} \\ \ln \left\{ ab^{ab} \left[\frac{S+y}{(1+ba)(y+w_{FP}-V)} \right]^{ab+1} \right\} > 0 & \text{for middle income} \\ 0 & \text{for high income} \end{cases}$$

As a consequence, the high-income worker with impure altruism ($c > 0$) will always prefer to work with the nonprofit. In contrast, for the cash-constrained workers (low and middle income), there exists an impure-altruism positive ceiling, below which they will chose to “earn to give” even though the market wages are the same in both types of firms. The reason is that they know their cash constraint make them unable to compensate the lack of labor donation by extra cash donation.

Second, if we exclude both warm glow for regular workers and labor donation: ($\Delta = V = 0$), the thresholds become:

$$\bar{c} = \begin{cases} 0 & \text{for low income} \\ \ln \left\{ ab^{ab} \left[\frac{S+y}{(1+ba)(y+w_{FP})} \right]^{ab+1} \right\} > 0 & \text{for middle income} \\ 0 & \text{for high income} \end{cases}$$

It follows that, without labor donation, the low-income workers with impure altruism ($c > 0$) will always prefer to work with the nonprofit. For middle-income motivated workers, the absence of labor donation makes threshold \bar{c} smaller than in the previous situation.

5. Conclusion

We consider workers who are motivated to make the world a better place. Most of the theory of labor markets instead considers “non-motivated” workers. They work to earn wages, but they are not intrinsically motivated by the output of their employer. We join recent studies in departing from that assumption. Much of recent analysis of motivated workers focuses on volunteers. The model we present describes motivated workers who have standard wage-paying jobs. The twist is that they gain utility from the output of a charitable organization, and, as a result, they might be willing to accept a lower wage working for a nonprofit than what they would require from a for-profit. When they do so, the worker is effectively donating time. The motivation here is not due simply to “warm glow,” but reflects a deeper altruistic impulse (Imas, 2014; Lilley & Slonim, 2014).

Even if she does not work for an environmental, anti-poverty, housing nonprofit, or similar charitable organization, the worker still wants to help, and she can do so by working at a standard for-profit firm and donating money. If her for-profit salary is high enough, she may in fact help the nonprofit organization more by donating money than by working for it directly. This is the gist of the “earning to give” principle championed by advocates of effective altruism.

We have focused on trade-offs between giving time and money. Andreoni (2006, p. 1252) concludes: “In sum, the literature on time and money contributions is in great flux, and there has yet to be a definitive study to address this gap.” Our goal has been to simplify in order to highlight particular mechanisms and relationships. Next steps will require greater complexity. For example, we use a utility function that, although standard, has specific features that could drive some results (such as the proportionality result that aligns the interests of the motivated worker and the nonprofit firm). Another simplification comes from the fact that we assume that the job market is in equilibrium, so that everybody is passive except for the motivated worker. This allows us to focus on the worker’s occupational choice. In a more general setting, the non-profit and for-profit may act actively and strategically.

Paradoxically, a byproduct of effective altruism and “earning to give” could be wage equalization across the for-profit and nonprofit sectors, especially for the high-end jobs for which the wage gap is large (Weisbrod, 1983; Preston 1990). The pay increase in nonprofits could indeed stem from the desire to attract or retain the skilled and most motivated employees (Jones, 2015). Since efficiency-conscious donors typically rank charities on the

basis of their overheads (Cochrane & Thornton, 2016; Meer, 2017), a wage increase would further hurt nonprofits relying on skilled workers.

Overall, our simple model emphasizes that altruism can deeply impact the occupational choices of pro-socially motivated workers. To them, higher wages are not only a way to increase their own consumption, but also a tool for enhancing their contribution to the greater good. We also show another paradox: that the presence of warm glow feelings can distort optimal choices from the perspective of nonprofits. In the light of the momentum effective altruism is gaining, further work could investigate the potential consequences of this new approach of occupational choice and the possible distortions to the occupational choices.

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Appendix A: Proof of Proposition 3

From Propositions 1 and 2 we have:

$$D_{CC}^* = \text{Max} \{0, X_1\}$$

$$D_{ETG}^* = \text{Max} \{0, X_2\}$$

where:

$$X_1 = \widetilde{w}_{NP} + \frac{-S+ba y}{1+ba}$$

and

$$X_2 = w_{NP} + \frac{-S+ba(y+\Delta)}{1+ba}$$

Moreover, the two wage gaps are expressed with reference to the for-profit market wage as:

$$w_{FP} = \widetilde{w}_{NP} + V = w_{NP} + \Delta$$

which yields:

$$X_1 = w_{FP} - V + \frac{-S+ba y}{1+ba} = w_{FP} + \frac{-S+ba y-(1+ba)V}{1+ba}$$

and:

$$X_2 = w_{FP} - \Delta + \frac{-S+ba(y+\Delta)}{1+ba} = w_{FP} + \frac{-S+ba y-\Delta}{1+ba}$$

Finally,

$$V \geq \Delta \Rightarrow (1+ba)V \geq \Delta.$$

It follows that $X_1 \leq X_2$.

QED

Appendix B: Proof of Proposition 4

If $w_{FP} \leq \frac{-S+ba y-\Delta}{1+ba}$ (low income):

$$C_{ETG}^* = y + w_{FP} \geq C_{CC}^* = y + w_{FP} - V$$

If $\frac{-S+ba y-\Delta}{1+ba} < w_{FP} \leq \frac{-S+ba y-(1+ba)V}{1+ba}$ (middle income):

$$C_{ETG}^* = \frac{S+y+\Delta}{1+ba} \geq C_{CC}^* = y + w_{FP} - V$$

because:

$$\begin{aligned} \frac{S+y+\Delta}{1+ba} - (y + w_{FP} - V) &= \frac{S-ba y+\Delta+(1+ba)V}{1+ba} - w_{FP} \\ &\geq \frac{S-ba y+\Delta+(1+ba)V}{1+ba} + \frac{-S+ba y-\Delta}{1+ba} = \Delta \geq 0 \end{aligned}$$

If $w_{FP} \geq \frac{-S+ba y-(1+ba)V}{1+ba}$ (high income):

$$C_{ETG}^* = \frac{S+y+\Delta}{1+ba} \geq C_{CC}^* = \frac{S+y}{1+ba}$$

$$C_{ETG}^* = \frac{S+y+\Delta}{1+ba} \geq C_{CC}^* = \frac{S+y}{1+ba}$$

We use the monotonicity of the log function to conclude that $\Delta \ln C^* \leq 0$

QED

Appendix C: Proof of Proposition 5

If $w_{FP} \leq \frac{-S+ba y-(1+ba)V}{1+ba}$ (low income):

$$Q_{CC}^* = (S - w_{FP} - V)^a \leq Q_{ETG}^* = (S - w_{FP} - \Delta)^a$$

If $\frac{-S+ba y-(1+ba)V}{1+ba} < w_{FP} \leq \frac{-S+ba y-\Delta}{1+ba}$ (middle income):

$$Q_{CC}^* = (S - w_{FP} + V)^a \text{ and } Q_{ETG}^* = \left(\frac{ba(y+S+\Delta)}{1+ba}\right)^a \geq Q_{CC}^*$$

$$\text{because: } S - w_{FP} + V \leq S - \frac{S-bay-(1+ba)V}{1+ba} = \frac{ba(y+S)+(1+ba)V}{1+ba} \leq \frac{ba(y+S+\Delta)}{1+ba}$$

If $w_{FP} \geq \frac{-S+ba y-\Delta}{1+ba}$ (high income):

$$Q_{CC}^* = \left(\frac{ba(y+S)}{1+ba}\right)^a \text{ and } Q_{ETG}^* = \left(\frac{ba(y+S+\Delta)}{1+ba}\right)^a \geq Q_{CC}^*$$

We use the monotonicity of log to derive that $\Delta \ln Q^* \leq 0$.

QED

Appendix D: Proof of Theorem 1

We have:

$$\Delta U = U_{CC} - U_{ETG} = \underbrace{\Delta \ln C^*}_{\leq 0} + b \underbrace{\Delta \ln Q^*}_{\leq 0} + c$$

Then it is obvious that:

$c = 0 \Rightarrow \Delta U \leq 0$, and the earning-to-give solution is dominant because it delivers higher utility to the motivated worker.

When $c > 0$, the compassionate-career solution will be chosen only if:

$$c \geq \bar{c} = -\Delta \ln C^* - b \Delta \ln Q^*$$

The proofs of Propositions 4 and 5 show that \bar{c} depends on the level of income of the motivated worker. Precisely we have:

$$\Delta \ln C^* = \ln C_{CC}^* - \ln C_{ETG}^* = \ln \frac{C_{CC}^*}{C_{ETG}^*} = \begin{cases} \ln \frac{y+w_{FP}-V}{y+w_{FP}} & \text{for low income} \\ \ln \frac{(1+ba)(y+w_{FP}-V)}{S+y+\Delta} & \text{for middle income} \\ \ln \frac{S+y}{S+y+\Delta} & \text{for high income} \end{cases}$$

and:

$$\Delta \ln Q^* = \ln Q_{CC}^* - \ln Q_{ETG}^* = \ln \frac{Q_{CC}^*}{Q_{ETG}^*} = \begin{cases} a \ln \frac{S-w_{FP}-V}{S-w_{FP}-\Delta} & \text{for low income} \\ a \ln \frac{(1+ba)(y+w_{FP}-V)}{ba(y+S+\Delta)} & \text{for middle income} \\ a \ln \frac{S+y}{S+y+\Delta} & \text{for high income} \end{cases}$$

We obtain the threshold value for impure altruism:

$$\bar{c} = -\Delta \ln C^* - b \Delta \ln Q^* = \begin{cases} \ln \frac{y+w_{FP}}{y+w_{FP}-V} + ab \ln \frac{S-w_{FP}-\Delta}{S-w_{FP}-V} & \text{for low income} \\ \ln \frac{S+y+\Delta}{(1+ba)(y+w_{FP}-V)} + ab \ln \frac{ba(y+S+\Delta)}{(1+ba)(y+w_{FP}-V)} & \text{for middle income} \\ \ln \frac{S+y+\Delta}{S+y} + ab \ln \frac{S+y+\Delta}{S+y} & \text{for high income} \end{cases}$$

$$= \begin{cases} \ln \left[\frac{y+w_{FP}}{y+w_{FP}-V} \left(\frac{S-w_{FP}-\Delta}{S-w_{FP}-V} \right)^{ab} \right] & \text{for low income} \\ \ln \left\{ ab^{ab} \left[\frac{S+y+\Delta}{(1+ba)(y+w_{FP}-V)} \right]^{ab+1} \right\} & \text{for middle income} \\ \ln \left[\left(\frac{S+y+\Delta}{S+y} \right)^{ab+1} \right] & \text{for high income} \end{cases}$$

QED