A Macroeconomic Model with Financial Panics

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1 The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Board or the Federal Reserve System.
What we do

- Incorporate banks and banking panics in simple macro model

- Broad goal:
  - Develop framework to understand dynamics of recent financial crisis

- Specific goals:
  - Characterize sudden/discrete nature of financial collapse in fall 2008
    - No observable large exogenous shock
    - Gorton (2010), Bernanke (2010): Bank runs at heart of collapse
  - Model credit boom preceding crisis
    - Optimistic beliefs before crisis (Bordalo et al (2017))
    - Increases susceptibility to runs
Motivation

1. GDP Growth and Credit Spreads

GDP Growth
BAA-10 Year Treasury Spread

2. Broker Liabilities

Lehman failure

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Model Overview

- Simple New Keynesian model with investment

- Banks intermediate funds between households and productive capital
  - Hold imperfectly liquid long term assets and issue short term debt →
  - Vulnerable to panic failure of depositors to roll over short term debt
    - Based on GK (2015) and GKP (2016)
    - In turn based on Cole/Kehoe(2001) self-fulfilling sovereign debt

- Households may directly finance capital, but less efficient at margin than banks
End of period capital $S_t$ vs. beginning $K_t$

$$S_t = \Gamma(l_t) + (1 - \delta)K_t$$

$\Gamma' > 0$, $\Gamma'' < 0$

$S_t \rightarrow K_{t+1}$:

$$K_{t+1} = \xi_{t+1}S_t$$

$\xi_{t+1} \equiv "capital quality"$ shock

$S_t^b$ intermediated by banks; $S_t^h$ directly held by households

$$S_t = S_t^b + S_t^h$$
Household and Bank intermediation

- Rate of return on intermediated capital

\[ R_{t+1}^b = \xi_{t+1} \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t} \]

- Utility cost to household of direct finance

\[ \varsigma(S_h^t, S_t) = \frac{\chi}{2} \left( \frac{S_h^t}{S_t} \right)^2 S_t \]

- Marginal rate of return on directly held capital

\[ R_{t+1}^h = \frac{1}{1 + \chi \frac{S_t}{S_t}} R_{t+1}^b \]

Where \( \chi \frac{S_t}{S_t} \) is the marginal cost of direct finance.
### NO BANK RUN EQUILIBRIUM

**BANKS**

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t S^b_t$</td>
<td>$D_t$</td>
</tr>
<tr>
<td>$N_t$</td>
<td></td>
</tr>
</tbody>
</table>

**CAPITAL**

$S_t$

**DIRECT CAPITAL HOLDING**

$Q_t S^h_t$

**HOUSEHOLDS**

### BANK RUN EQUILIBRIUM

**CAPITAL**

$S_t$

$Q^*_t S_t$

**HOUSEHOLDS**
Bank Decision Problem

- **Objective**

  \[ V_t = E_t \Lambda_{t,t+1}[(1 - \sigma)n_{t+1} + \sigma V_{t+1}] \]

  \( \sigma \equiv \) exogenous survival probability

- **Net worth** \( n_t \) accumulated via retained earnings - no new equity issues

  \[ n_{t+1} = R^k_{t+1} Q_t s^b_t - \overline{R}_t d_t \text{ if no run} \]

  \[ = 0 \text{ if run} \]

- **Balance sheet**

  \[ Q_t s^b_t = d_t + n_t \]
Deposit Contract

\( \bar{R}_t \equiv \text{deposit rate}; \ R_{t+1} \equiv \text{return on deposits} \)

\( p_t \equiv \text{run probability}; \ x_{t+1} < 1 \equiv \text{recovery rate} \)

- Deposit contract: (One period)

\[
R_{t+1} = \begin{cases} 
\bar{R}_t \text{ with prob. } 1 - p_t \\
x_{t+1} \bar{R}_t \text{ with prob. } p_t
\end{cases}
\]

- Recovery rate:

\[
x_{t+1} = \frac{\xi_{t+1} \left[ Z_{t+1} + (1 - \delta) Q^*_{t+1} \right]}{\bar{R}_t D_t} S^b_t
\]
Perfect markets:

Banks issue deposits until:

\[ E_{t} \Lambda_{t,t+1} \{ R_{t+1}^k - R_{t+1} \} = 0 \]

⇒ Leverage constraints do not arise
⇒ Financial panics cannot arise

Limits to arbitrage:

Occasionally binding leverage constraints⇒

\[ E_{t} \Lambda_{t,t+1} \{ R_{t+1}^k - R_{t+1} \} > 0 \]

Bank runs possible: extreme increases in \( E_{t} \Lambda_{t,t+1} \{ R_{t+1}^k - R_{t+1} \} \)
Moral Hazard Problem:

- After banker borrows funds at $t$, it may divert fraction $\theta$ of assets for personal use.
- If bank does not honor its debt, creditors can
  - recover the residual funds and
  - shut the bank down.

$\Rightarrow$ Incentive constraint (IC)

$$\theta Q_t s^b_t \leq V_t$$
Can show \( V_t = \psi_t n_t \) with \( \psi_t \geq 1 \) and increasing in \( E_t\{R_{t+1}^k - R_{t+1}\} \)

Combine with \( IC \to \) endogenous leverage constraint:

\[
Q_t s_t^b \leq \bar{\phi}_t n_t
\]

\[
\bar{\phi}_t = \frac{\psi_t}{\theta}
\]

Note:

- \( n_t \leq 0 \Rightarrow \) bank cannot operate (key for run equilibria)
- \( E_t\{R_{t+1}^k - R_{t+1}\} \) countercyclical \( \Rightarrow \bar{\phi}_t \) countercyclical.
Aggregation: No Run Case

Homogeneity: \( \phi_t \equiv \frac{Q_t s^b_t}{n_t} \) and \( \bar{\phi}_t \) independent of bank-specific factors

- Aggregate leverage constraint

\[
Q_t s^b_t \leq \bar{\phi}_t N_t
\]

- Aggregate net worth

\[
N_t = \sigma [ (R^k_t - R_t) \phi_{t-1} + R_t ] N_{t-1} + \zeta S_{t-1}
\]

- Absent runs, conventional financial accelerator with non-linearity
Bank Runs

- Self-fulfilling "bank run" equilibrium (i.e. rollover crisis) possible if:
  - A depositor believes that if other households do not roll over their deposits, the depositor will lose money by rolling over.
  - Condition met iff banks’ net worth $n_t$ goes to zero if others run
    - $n_t = 0 \rightarrow$ banks cannot operate
Conditions for Bank Run Equilibrium (BRE)

- Run equilibrium exists at $t + 1$ if

$$\xi_{t+1} \left( Z_{t+1}^* + (1 - \delta) Q_{t+1}^* \right) S^b_t < D_t \bar{R}_t \quad (1)$$

where $Q_{t+1}^* \equiv$ liquidation price:

$$Q_t^* = E_t \{ \Lambda_{t, t+1} \xi_{t+1} (Z_{t+1} + (1 - \delta) Q_{t+1}) \} - \chi \frac{S^h_t}{S_t}$$

evaluated at $\frac{S^h_t}{S_t} = 1$

- $\iota_{t+1} \equiv$ sunpot variable; if $\iota_{t+1} = 1$ depositors panic when run possible

- Run occurs if (i) equation (1) is satisfied and (ii) $\iota_{t+1} = 1$
Run Probability $p_t$

- Assume sunspot occurs with probability $\kappa$.

→ The time $t$ probability of a run at $t + 1$ is

$$p_t = \Pr_t \left\{ \xi_{t+1} \left( Z_{t+1}^* + (1 - \delta) Q_{t+1}^* \right) S_t^b < D_t \bar{R}_t \right\} \cdot \kappa$$

$\Leftrightarrow$

$$p_t = \Pr_t \left\{ \xi_{t+1} \left( Z_{t+1}^* + (1 - \delta) Q_{t+1}^* \right) < \frac{D_t \bar{R}_t}{S_t^b} \right\} \cdot \kappa$$

→ Higher leverage ratios $\frac{D_t \bar{R}_t}{K_t^b}$ increase run probability
Production, resource constraint and $Q$ relation for investment

\[ Y_t = AK_t^\alpha L_t^{1-\alpha} \]
\[ Y_t = C_t + I_t + G \]
\[ Q_t = \Phi(I_t) \]

Monopolistically comp. producers with quadratic costs of nominal price adjustment (Rotemberg)

- Adjust output to meet demand
- New Keynesian Phillips curve relating inflation to marginal cost

Monetary policy: simple Taylor rule

\[ R_t^n = \frac{1}{\beta} \left( \frac{P_t}{P_{t-1}} \right)^{\kappa_\pi} (\Theta_t)^{\kappa_y} \]
### Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Impatience</td>
<td>.99</td>
<td>Risk Free Rate</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>Risk Aversion</td>
<td>2</td>
<td>Literature</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Frish Elasticity</td>
<td>2</td>
<td>Literature</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of subst across varieties</td>
<td>11</td>
<td>Markup 10%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
<td>.33</td>
<td>Capital Share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>.025</td>
<td>$\frac{I}{K} = .025$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of q to i</td>
<td>.25</td>
<td>Literature</td>
</tr>
<tr>
<td>$a$</td>
<td>Investment Technology Parameter</td>
<td>.53</td>
<td>$Q = 1$</td>
</tr>
<tr>
<td>$b$</td>
<td>Investment Technology Parameter</td>
<td>-.83%</td>
<td>$\frac{I}{K} = .025$</td>
</tr>
<tr>
<td>$G$</td>
<td>Government Expenditure</td>
<td>.45</td>
<td>$\frac{G}{Y} = .2$</td>
</tr>
<tr>
<td>$\rho^{ir}$</td>
<td>Price adj costs</td>
<td>1000</td>
<td>Slope of Phillips curve .01</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>Policy Response to Inflation</td>
<td>1.5</td>
<td>Literature</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>Policy Response to Output</td>
<td>.5</td>
<td>Literature</td>
</tr>
</tbody>
</table>

#### Financial Intermediation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Banker Survival rate</td>
<td>.93</td>
<td>Leverage $\frac{QS^b}{N} = 10$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>New Bankers Endowments as a share of Capital</td>
<td>.1%</td>
<td>$% \Delta I$ in crisis $\approx 35%$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Share of assets divertible</td>
<td>.22</td>
<td>Spread Increase in Crisis = 1.5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>HH Intermediation Costs</td>
<td>.61</td>
<td>$\frac{S^b}{S} = .33$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>HH Intermediation Costs</td>
<td>.105</td>
<td>$ER^b - R = 2%$ Annual</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Sunspot Probability</td>
<td>.15</td>
<td>Run Probability 4% Annual</td>
</tr>
<tr>
<td>$\sigma(e^\xi)$</td>
<td>std of innovation to capital quality</td>
<td>.5%</td>
<td>std Output (C+I)</td>
</tr>
<tr>
<td>$\rho^\xi$</td>
<td>serial correlation of capital quality</td>
<td>.7</td>
<td>std Investment</td>
</tr>
</tbody>
</table>
Response to a Capital Quality Shock: No Run Case
Response to a Sequence of Shocks: Run VS No Run

RUN (Run Threshold Shock and Sunspot) — NO RUN (Run Threshold Shock and No Sunspot)

- **Capital Quality**
- **Run Probability**
- **Bank Net Worth**
- **Leverage Multiple: $\phi$**
- **Investment**
- **Output**
- **Excess Return: $ER^b_{R^{free}}$**
- **Policy Rate**
- **Inflation**

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Financial Crisis: Model vs Data

Shocks:
-0.2%  -0.5%  -0.4%  -0.6%  -0.6%
Threshold:
-0.9%  -0.8%  -0.7%  -0.7%  -0.6%

1. Investment

Bear Stearns
Lehman Brothers

2. XLF Index and Net Worth

3. Spreads (AAA-Risk Free)

4. GDP

5. Labor (hours)

6. Consumption

NOTE: The data for GDP, Investment, and Consumption are computed as logged deviations from trend where the trend is the CBO potential hours worked. The XLF Index data is computed as the percent deviation from its 2007q3 level.

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Boom leading to the bust: news driven optimism

- Capital quality:

\[ \xi_{t+1} = \rho \xi_t + \epsilon_{t+1} \]

- At \( t = 0 \) bankers learn that unusually large realization of \( \epsilon_{t+1} \) of size \( B > 0 \) will happen at \( t^B \in \{1, \ldots, T\} \) with prob. \( P_0^B < 1 \)

- \( \Pr_0\{t^B = t\} \) is a truncated Normal (discrete approx.)

- Agents update \( \Pr_t \) and \( \overline{P}_t^B \) by observing \( \epsilon_t \)

- Prob. at \( t \) of shock at \( t + 1 \) is \( \Pr_t\{t^B = t + 1\} \cdot \overline{P}_t^B \)

- Implies forecast errors in line with evidence, e.g. Bordalo et al 2017
Financial Crisis After Credit Boom: Model vs Data

1. Investment

2. XLF Index and Net Worth

3. Spreads (AAA-Risk Free)

4. GDP

5. Labor (hours)

6. Consumption

Shocks: -0.2% -0.4% -0.3% -0.5% -0.0%
Threshold: -0.1% -0.1% 0.0% -0.0% -0.0%

NOTE: The data for GDP, Investment, and Consumption are computed as logged deviations from trend where the trend is the CBO potential hours worked. The XLF Index data is computed as the percent deviation from its 2007q3 level.
Forecast Errors: AAA-Treasury (4-Quarters Ahead)

Error (Next 4Q Average) = Actual - Forecast

Data (Bordalo-Gennaioli-Shleifer) vs. Model

Bear Stearns
Lehman Brothers

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Conclusion

- Incorporated banking sector with conventional macro model
  - Banks occasionally exposed to self-fulfilling rollover crises
  - Crises lead to significant contractions in real economic activity

- Model captures qualitatively and quantitatively
  - Nonlinear dimension of financial crises
  - The broad features of the recent recent collapse
  - Credit boom preceding crisis

- Next steps:
  - Macroprudential policy (Run Externality)
  - Lender-of-last resort policies
Run Equilibrium Threshold

\[ \frac{\xi_{t+1}(Z_{t+1} + (1 - \delta)Q^*_t)}{R_{t+1}} \]

No Run-Equilibrium Possible

Negative Capital Quality Shock

Run-Equilibrium Possible
We can simplify existence condition for BRE:

\[ x_t = \frac{R_t^{b^*}}{R_t} \cdot \frac{\phi_{t-1}}{\phi_{t-1} - 1} < 1 \]

with

\[ R_t^{b^*} = \frac{\xi_t[Z_t + (1-\delta)Q_t^*]}{Q_{t-1}}, \quad \phi_{t-1} = \frac{Q_{t-1}S_{t-1}^b}{N_{t-1}} \]

- Likelihood BRE exists decreasing in \( Q^*(\cdot) \) and increasing in \( \phi_{t-1} \)
- \( \phi_{t-1} \) countercyclical \( \rightarrow \) likelihood BRE exists is countercyclical.
Run Equilibrium Threshold

Negative Capital Quality shock

No Run-Equilibrium Possible

Run-Equilibrium Possible

\( \frac{R_t^{\mu}}{R_t} \)

\( \phi_t \)
How we differ

- Conventional financial accelerator/credit cycle models
  (e.g. Gertler/Kiyotaki 2011)
  - Mutual feedback between borrower balance sheets and real activity
  - Local approximations $\rightarrow$ dynamics linear

- Models with occasionally binding balance sheet constraints
  (e.g. Brunnermeier/Sannikov 2014, He/Krishnamurthy, 2016)
  - Moving from unconstrained to constrained region $\Rightarrow$ nonlinear contraction

- This paper: both occasionally binding constraints and bank runs
  - Runs more significant source of non-linearity
  - Richer macro model
Response to a Sequence of Shocks in Flex Price Economy: Run VS No Run

- Capital Quality
- Run Probability
- Bank Net Worth
- Leverage
- Investment
- Output
- Excess Return: ER^B-R^free
- Natural Rate
- Consumption

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