Topic 2: Part 3

Introducing Heterogeneity and Borrowing Constraints:

Implications for Output Dynamics and the Liquidity Trap

Mark Gertler NYU

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Debt, Deleveraging and the Liquidity Trap (Eggertsson/Krugman)

- Objective: introduce heterogeneity and lending and borrowing in simple NK model
  - Allow for financial constraints that impede credit flow
  - Illustrate how tightening of financial constraints may reduce aggregate demand,
  - By doing, may reduce the natural rate of interest, possibly moving the economy into a liquidity trap and recession
  - Illustrate how the deleveraging process (drawing down of debt) can cause the downturn to persist.

- Motivation: tightening of borrowing constraints on households played an important role in Great Recession
  - Decline housing prices limited ability to obtain credit
  - Financial institutions that experienced losses also tightened lending terms.
Setup

- Baseline: NK model with consumption goods only
- Two types of agents:
  - Saver: consumes $C_t^s$ and lends the amount $D_t$ in capital market.
    * Discount factor of $\beta$
  - Borrower: consumes $C_t^b$ and borrows $D_t$
    * Discount factor of $\gamma < \beta$ (motive for borrowing)
    * Faces borrowing constraint $R_{t+1}D_t \leq \overline{D}_t$
- For simplicity we assume borrowers get the fraction $\nu$ of output $Y_t$ and savers the fraction $1 - \nu$
  - Goal is to derive IS curve, not complete model
- We also restrict attention to real debt, but discuss the implications of nominal debt and deflation (which raises real debt burdens).
Borrower Behavior

No uncertainty, abstract from labor supply - deterministic problem

- **Objective**

  \[
  \max_{C_t^b, D_t} E_t \sum_{i=0} \gamma^i \log C_{t+i}^b
  \]

- **Budget constraint**

  \[
  C_t^b = vY_t - R_tD_{t-1} + D_t
  \]

- **Borrowing constraint**

  \[
  R_{t+1}D_t \leq \bar{D}_t
  \]
Borrower’s Decision Problem

- Bellman equation

\[ V_t(R_tD_{t-1}) = \max_{C_t, D_t} (\log C_t + E_t\{\beta V(R_{t+1}D_t)\}) \]

subject to

\[ C_t^b \quad = \quad vY_t - R_tD_{t-1} + D_t \]
\[ R_t+1D_t \quad \leq \quad D_t \]

\(\Omega_t \equiv \) Lagrange multiplier on borrowing constraint (i.e. the shadow value of increasing the debt limit)

- First order necessary condition for consumption/saving

\[ \frac{1}{C_t^b} \quad = \quad R_{t+1}\{E_t\{\gamma\frac{1}{C_t^b}\} + \Omega_t\} \]
Solution

- If borrowing constraint does not bind (i.e. $\Omega_t = 0$)

$$\frac{1}{C^b_t} = R_{t+1} E_t \{ \gamma \frac{1}{C^b_{t+1}} \}$$

- If constraint binds (i.e. $\Omega_t > 0$)

$$C^b_t = v Y_t - \overline{D}_{t-1} + \overline{D}_t / R_{t+1}$$

- Note:
  - Constraint more likely to bind, the lower the discount factor $\gamma$
  - Tightening the borrowing limit $\overline{D}_t$ reduces $C^b_t$
  - Conversely, lower inherited debt $\overline{D}_{t-1}$ raises $C^b_t$. 

Saver Behavior

• Objective

\[ \max_{C_t^s, D_t} E_t \sum_{i=0} \beta^i \log C_{t+i}^s \]

• Budget constraint

\[ C_t^b = (1 - \nu)Y_t + R_t D_{t-1} - D_t \]

→

First order necessary condition

\[ \frac{1}{C_t^b} = R_{t+1} E_t \left\{ \beta \frac{1}{C_{t+1}^b} \right\} \]

Note \( \beta > \gamma \rightarrow \) stronger incentive to save than for borrower
Equilibrium (taking output as given for now)

- Resource constraint:

\[ Y_t = C_t = C^s_t + C^b_t \]

- Saver behavior

\[ \frac{1}{C^s_t} = R_{t+1}E_t\{\beta \frac{1}{C^s_{t+1}}\} \]

- Borrower behavior (assuming borrowing constraint is binding)

\[ C^b_t = vY_t - \overline{D}_{t-1} + \overline{D}_t/R_{t+1} \]
Deterministic Steady State

- From saver behavior

\[
\frac{1}{C^b} = R\beta \frac{1}{C^b} \rightarrow \\
1 = R\beta
\]

- From borrower

\[
C^b = vY - D + D/R \rightarrow \\
C^b = vY - \frac{R - 1}{R}D
\]

- From resource constraint and borrower

\[
C^s = Y - C^b \\
= (1 - \nu)Y + \frac{R - 1}{R}D
\]

Given $Y$, $C^b$ varies inversely with $D$ and $C^s$ positively.
The Short Run, Deleveraging and the Liquidity Trap

- Derive IS curve (a relation for $Y$ conditional on $R$) from saver’s Euler equation

\begin{align*}
C_t^s &= (\beta R_{t+1})^{-1} E_t C_{t+1}^s \rightarrow \\
Y_t - C_t^b &= (\beta R_{t+1})^{-1} E_t (Y_{t+1} - C_{t+1}^b) \rightarrow \\
Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} + C_t^b - (\beta R_{t+1})^{-1} C_{t+1}^b
\end{align*}

\begin{align*}
Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} \\
&\quad + \nu Y_t - \overline{D}_{t-1} + \overline{D}_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t (\nu Y_{t+1} - \overline{D}_t + \overline{D}_{t+1} / R_{t+2})
\end{align*}

\begin{align*}
Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} \\
&\quad + \frac{1}{1 - \nu} \left[ -\overline{D}_{t-1} + (1 + \beta^{-1}) \overline{D}_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t (\overline{D}_{t+1} / R_{t+2}) \right]
\end{align*}
IS Curve with Debt Constraints

\[
Y_t = (\beta R_{t+1})^{-1} E_t Y_{t+1} \\
+ \frac{1}{1 - \nu} \left[ -D_{t-1} + (1 + \beta^{-1}) D_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t (D_{t+1} / R_{t+2}) \right]
\]

\( \frac{1}{1 - \nu} \) is multiplier arises because \( C_t^b \) depends on \( Y_t \)

- Debt constraint affects position of IS curve. Given \( R_{t+1}, R_{t+2} \)
  - Increased debt overhang reduces output \( D_{t-1} \uparrow \rightarrow C_t^b \downarrow \rightarrow Y_t \downarrow \) (\( \frac{1}{1 - \nu} \) is multiplier effect due to effect of \( Y_t \) on \( C_t^b \)).
  - Tightening of borrower limit reduces output \( D_t \downarrow \rightarrow C_t^b \downarrow \) and \( C_t^s \downarrow \) (the latter because \( C_{t+1}^s \downarrow \)) \rightarrow Y_t \downarrow
"Deleveraging" Shock and the Liquidity Trap

• Determination of natural rate of interest $R_{t+1}^*$:

$$Y_t^* = (\beta R_{t+1}^*)^{-1} E_t Y_{t+1}^* + \frac{1}{1 - \nu} [-\overline{D}_{t-1} + (1 + \beta^{-1}) \overline{D}_t/R_{t+1}^* - \overline{D}_t+1/\beta R_{t+1}^* R_{t+2}^*]$$

where $Y_t^*$ ≡ natural rate of output

• Deleveraging shock ≡ tightening of borrowing limit which forces a reduction in leverage: → drop in $\overline{D}_t$

• Drop in $\overline{D}_t$ induces drop in $R_{t+1}^*$:
  – Intuitively: $\overline{D}_t \downarrow$ induces drop in spending. $R_{t+1}^*$ must fall to induce an increase in saver spending to make $Y_t = Y_t^*$.

• If the drop is large enough, $R_{t+1}^*$ goes below unity → ZLB binds.

• With nominal debt, a fall in the price level raises the inherited real debt burden $\overline{D}_{t-1} \rightarrow$ spiral of output contraction and deflation
Some Issues

- Borrowing constraint exogenous

- Debt and debt dynamics exogenous (driven by exogenous variation in debt constraint).
  - Except when debt is in nominal terms, i.e., $D_t = \frac{D^n_t}{P_t}$ where $D^n_t$ is the nominal value of the debt. As the economy weakens, the price level falls, raising real debt burdens. This induces a further decline in output, and so on.

- An MPC of unity for constrained borrowers seems unrealistic. Borrowers may use some of extra income to pay the down debt.
  - Will happen in an environment with uncertainty as to whether the constraint will be binding.
  - Can have "precautionary" saving (i.e. building up buffer of liquid assets) to limit impact of constraint if it becomes binding. (Will also have precautionary saving with transitory income uncertainty).
- If alternative saving vehicles are available, tightening of household borrower constraints will not push natural rate to zero
  - With borrowers constrained, savers will substitute to these alternative assets with modest declines in real rates.
  - Unless there are frictions in supplying funds to these sectors.