Topic 4

Investment

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Investment

Motivation: Durable goods central to business cycles and transmission of monetary policy, but missing in simple NK model.

We now add investment in the form of producer durables (capital)

   Straightforward to add residential investment and consumer durables

   Working hypothesis: dynamics of producer durables similar to aggregate including housing and consumer durables.

Three steps:
1. Partial equilibrium model of investment: Tobin’s "Q" Theory
2. Integrate "Q" theory investment into NK model
3. Explore implications for fluctuations and monetary policy, etc.
4. Describe additional modifications needed to match data.
Tobin’s Q Theory

Relates investment rate to its "Q" value: i.e. the ratio of the shadow value of a unit of capital within the firm to its replacement cost.

Key assumption: convex costs to a firm of adjusting it’s capital stock.

→ The value of a unit of installed capital can exceed replacement cost.

→ $K_t$ is a predetermined state of the firm

Let $p_t^I \equiv$ replacement cost of capital.

Then the cost of acquiring $I_t$ units of capital is given by

$$p_t^I (I_t + \frac{1}{2}c(\frac{I_t}{K_t})^2 K_t)$$

Where $p_t^I \frac{1}{2}c(\frac{I_t}{K_t})^2 K_t \equiv$ adjustment costs (which we assume are in units of new capital).

Note we assume constant returns to scale.
Tobin’s Q Theory: Optimization Problem

Production

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]

Capital

\[ K_{t+1} = I_t + (1 - \delta)K_t \]

Profits

\[ \Pi_t = Y_t - W_t L_t - p_t^I I_t - \frac{1}{2} p_t^I c(I_t/K_t)^2 K_t \]

Objective: Let \( \Lambda_{t,t+i} \equiv \) stochastic discount factor

\[ \max_{Y_t, L_t, I_t K_{t+1}} E_t \left\{ \sum_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} \right\} \]
Optimization Problem (con’t)

Bellman’s equation.

\[
V(K_t, A_t) = \max_{Y_t, L_t, I_t K_{t+1}} \Pi_t + E_t \{ \Lambda_{t,t+1} V(K_{t+1}, A_{t+1}) \}
\]

subject

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha}
\]

\[
K_{t+1} = I_t + (1 - \delta)K_t
\]

\[
\Pi_t = Y_t - W_t L_t - p_t^I I_t - \frac{1}{2} p_t^I c \left( \frac{I_t}{K_t} \right)^2 K_t
\]
Optimization Problem (con’t)

Eliminating the constraints:

\[
V(K_t, A_t) = \max_{L_t, I_t} A_t K_t \alpha L_t^{1-\alpha} - W_t L_t - p_t(I_t + \frac{1}{2}c(I_t^2/K_t))
\]

\[
+ E_t\{\Lambda_{t,t+1} V(I_t + (1 - \delta)K_t, A_{t+1})\}
\]

\[
(1 - \alpha) \frac{Y_t}{L_t} = W_t
\]

\[
p_t(I_t + cI_t/K_t) = E_t\{\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1})\}
\]

\[
\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1}) \equiv \text{discounted marginal value of a unit capital}
\]
Tobin’s Q Theory of Investment

Tobin’s Q: ratio of discounted marginal value of installed capital to its replacement cost

\[ Q_t = \frac{E_t\{\Lambda_{t,t+1}V_1(K_{t+1}, A_{t+1})\}}{P_t} \]

Rearranging fonc for investment

\[ \frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1) \]

→Investment depends positively on the difference between \( Q_t \) and unity

Sensitivity depends inversely on adjustment cost parameter \( c \)

Note it is the marginal adjustment cost that yields a wedge between \( Q_t \) and unity.

Otherwise firms would invest to the point where \( Q_t = 1 \)
To complete solution, need to find an expression for shadow value $\frac{E_t \{ A_{t,t+1} V_1(K_{t+1}, A_{t+1}) \}}{P_t}$
Tobin’s Q Theory of Investment (con’t)

Substituting for \( Q_t \):

\[
\frac{I_t}{K_t} = \frac{1}{c} \left( \frac{E_t \{ \Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1}) \}}{p_t} \right) - 1
\]

Need expression for \( V_1(K_{t+1}, A_{t+1}) \)

Envelope condition:

\[
V_1(K_t, A_t) = \alpha \frac{Y_t}{K_t} + \frac{1}{2} p_t c \left( \frac{I_t}{K_t} \right)^2 + (1 - \delta) E_t \{ \Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1}) \}
\]

\[
V_1(K_t, A_t) = E_t \{ \sum_{i=0}^{\infty} \Lambda_{t,t+i} (1 - \delta)^i [\alpha \frac{Y_{t+i}}{K_{t+i}} + \frac{1}{2} p_t c \left( \frac{I_{t+i}}{K_{t+i}} \right)^2] \}
\]

\[
E_t \{ \Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1}) \} = E_t \{ \sum_{i=1}^{\infty} \Lambda_{t,t+i} (1 - \delta)^{i-1} [\alpha \frac{Y_{t+i}}{K_{t+i}} + \frac{1}{2} p_t c \left( \frac{I_{t+i}}{K_{t+i}} \right)^2] \}
\]

= discounted sum of rental earnings and savings on adjustment costs.
Marginal vs. Average Q:

Marginal $Q$ is not directly observable

Need series on expected rental earnings and adjustment costs

However, with constant returns, marginal $Q = \text{average } Q$

Average $Q$ measured as ratio of market value to book value of capital.

Let $q_t = V_1(K_t, A_t)$. Then

$$q_t = \alpha \frac{Y_t}{K_t} + \frac{1}{2} p_t I_t c \left( \frac{I_t}{K_t} \right)^2 + (1 - \delta) E_t \{ \Lambda_{t+1} q_{t+1} \}$$

Multiply each side by $K_t$:

$$q_t K_t = \alpha Y_t + \frac{1}{2} p_t I_t c \left( \frac{I_t}{K_t} \right)^2 K_t + (1 - \delta) E_t \{ \Lambda_{t+1} q_{t+1} K_t \}$$

Since $(1 - \delta)K_t = K_{t+1} - I_t$:

$$q_t K_t = \alpha Y_t + \frac{1}{2} p_t I_t c \left( \frac{I_t}{K_t} \right)^2 K_t - E_t \{ \Lambda_{t+1} q_{t+1} I_t \} + E_t \{ \Lambda_{t+1} q_{t+1} K_{t+1} \}$$
Marginal vs. Average Q (con’t)

Given optimality condition \( p_t I (1 + c \frac{I_t}{K_t}) = E_t \{ \Lambda_{t,t+1} q_{t+1} \} \rightarrow \)

\[
p_t I (I_t + c(\frac{I_t}{K_t})^2) = E_t \{ \Lambda_{t,t+1} q_{t+1} I_t \}
\]

\[
q_t K_t = \alpha Y_t - p_t I (I_t + \frac{1}{2} c(\frac{I_t}{K_t})^2 K_t) + E_t \{ \Lambda_{t,t+1} q_{t+1} K_{t+1} \}
\]

\[
= \Pi_t + E_t \{ \Lambda_{t,t+1} q_{t+1} K_{t+1} \}
\]

\[
= E_t \{ \sum_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} \}
\]

given \( Y_t - W_t L_t = Y_t - (1 - \alpha)(Y_t/L_t) L_t = \alpha Y_t \)

Dividing thru by \( K_t \rightarrow \) marginal value of capital equals average value

\[
q_t = E_t \{ \sum_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} \} / K_t
\]
Investment and Average $Q$

$$\frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1)$$

$$Q_t = \frac{E_t\{\Lambda_{t,t+1}q_{t+1}\}}{p_t}$$

$$E_t\{\Lambda_{t,t+1}q_{t+1}\} = E_t\{\sum_{i=1}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} / K_{t+1}\}$$

→ marginal $Q = \text{average } Q$

$$Q_t = \frac{E_t\{\sum_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i}\}}{p_t K_{t+1}}$$

$Q$ is thus measured by the ratio of market value to replacement cost

(Requires constant returns, including CRS in adjustment costs)
Investment and Average Q (con’t)

\[ \frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1) \]

with

\[ Q_t = \frac{E_t \{ \sum_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} \}}{p_t K_{t+1}} \]

Note: \( Q_{t+1} \) summarizes the effect on investment of both expected future cash flows and discount rates (via \( \Lambda_{t,t+i} \))

Strong implication: \( Q_{t+1} \) should be a "sufficient statistic" for investment.

In practice, stock market measures of \( Q_t \) work poorly in explaining \( I_t \)

"Fundamental" measures based on forecasts of future earnings work better

Liquidity variables (cash flow, credit spreads) add explanatory power.
Adding Investment to the NK model

We now incorporate investment based on Tobin’s Q theory

We do so in a way that facilitates aggregation

Internal versus external adjustment costs

Internal: at the firm level (as in the previous partial equilibrium model)

External: aggregate increasing marginal costs in the production of new capital goods

Disadvantage of internal: capital stock is a predetermined state for each firm - complicates aggregation.

→ External adjustment costs typically standard

→ Generates "Q" behavior of investment.
Environment

• Representative household:
  – Consumes final good $C_t$, supplies labor $L_t$ at wage $W_t/P_t$
  – Saves in the form of:
    * Capital $K_t$, which it rents to firms at rate $Z_t$
    * One period private nominal bonds $B_t$ earning net nominal rate $r^n_t$.
    * Real money balances $M_t/P_t$,

• Three types of firms:
  – Final good producers: competitors
    * Produce output $Y_t$ using intermediate goods $Y_t(f)$.
  – Intermediate good firms: monopolistic competitors
    * Produce a differentiated product $Y_t(f)$ using capital $K_t(f)$ and labor $L_t(f)$. Set prices $P_t(f)$ on staggered basis.
Environment (con’t)

● Types of firms (con’t)
  – Capital producers: competitors
    * Use final output to make new capital which they sell at price $Q_t$
    * Investment involves adjustment costs → "Tobin’s Q" relation for investment along with variable price of capital.

● Central bank and government conducts monetary policy and fiscal policy.

● Frictionless financial markets
  – Monopolistic competition and price rigidities only distortions from first best.

Decision problems of households, final good firms and intermediate goods firms are essentially the same as in the baseline model with consumption goods only.

Hence we turn to the problem of capital producers
Capital Producers

- A representative capital producer invests $I_t$ units of final output and rents $K_t$ units of capital (after use in output production) to produce $J_t$ units of new capital.

- The technology for producing new capital goods is given by

$$J_t = I_t - \frac{1}{2}c\left(\frac{I_t}{K_t} - \delta\right)^2 \cdot K_t$$

- $\frac{1}{2}c\left(\frac{I_t}{K_t} - \delta\right)^2 \cdot K_t \equiv$ quadratic costs of adjusting capital stock (after depreciation $\delta$).

- The capital producer sells new capital to households at the market price $Q_t$.

- Can express maximization problem as choose $I_t$ to solve

$$\max Q_t J_t - I_t$$

s.t. production function for $J_t$

- Ignore $K_t$ since rental rate for use in investment is zero in a neighborhood of the ss (as we show).
Investment and $Q_t$

- FONC conditions for $I_t \rightarrow$

\[
\frac{I_t}{K_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q_t})
\]

- In steady state with zero growth, net investment, $\frac{I_t}{K_t} - \delta = 0; \rightarrow Q_t = 1$
- Outside ss, $\frac{I_t}{K_t} - \delta$ varies positively with $Q_t$
- Sensitivity of $\frac{I_t}{K_t}$ to $Q_t$ depends inversely on $c$.

- Marginal product of capital (for producing investment goods).

\[
\frac{\partial J_t}{\partial K_t} = \frac{1}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 \approx 0 \text{ near steady state}
\]

\[\rightarrow\] rental rate on capital $\approx 0$ near steady state.

- Can ignore capital
Resource constraints

- Income and expenditure

\[ Y_t = C_t + I_t + G_t \]

- Evolution of capital

\[ K_{t+1} = I_t - \frac{1}{2} c \left( \frac{I_t}{K_t} - \delta \right)^2 \cdot K_t + (1 - \delta) K_t \]
Monetary and Fiscal Policy

- The central bank sets the nominal interest rate according to the following simple feedback rule:

\[
1 + r_t^n = \max\{(1 + r) \left( \frac{P_t}{P_{t-1}} \right)^{\phi_{\pi}} \left( \frac{Y_t}{Y^*_t} \right)^{\phi_y} e^{\epsilon_t}, 1\}
\]  

(1)

where \(Y^*_t\) \(\equiv\) natural (i.e. flexible price equilibrium) level of output, with \(\phi_{\pi} > 1\) and \(\phi_y > 0\) and \(1 + r\) is the zero inflation steady state real interest rate, equal to the nominal rate.

- Fiscal policy

\[
G_t = G
\]

- Government budget constraint:

\[
G_t = T_t + \frac{M_t - M_{t-1}}{P_t}
\]  

(2)
Equilibrium

- An equilibrium is defined as an allocation \((Y_t, L_t, C_t, I_t, K_{t+1})\) and a price system \((Z_t, W_t, P_t, P_t^0, r_t^n, Q_t, \mu_t)\) such that all agents are maximizing subject to their respective constraints, all markets clear, and all resource constraints are satisfied, given \(P_{t-1}, A_t,\) and \(K_t\).

- In practice, it is convenient to express the equilibrium as the vector \((Y_t, C_t, I_t, N_t, P_t, P_t^0, R_t^n, Q_t, \mu_t, K_{t+1})\) that satisfies a system of 8 equations, given the predetermined states \(P_{t-1}, A_t,\) and \(K_t\) and \(\mu = \frac{1}{1-1/\varepsilon}\) (10 unknowns, 10 equations).

- It is useful to group the equations into aggregate demand, aggregate supply and policy blocks.
Aggregate Demand

output, consumption, investment, arbitrage:

\[ Y_t = C_t + I_t + G \]

\[ C_t = E_t \left\{ \left[ (1 + r^n_t) \frac{P_t}{P_{t+1}} \beta \right]^{-\sigma} C_{t+1} \right\} \]

\[ \frac{I_t}{K_t} = \delta + \frac{1}{c} \left( 1 - \frac{1}{Q_t} \right) \]

\[ E_t \left\{ \Lambda_{t,+1} (1 + r^n_t) \frac{P_t}{P_{t+1}} \right\} = E_t \left\{ \Lambda_{t,+1} \left( \frac{Z_t + (1-\delta)Q_{t+1}}{Q_t} \right) \right\} \]

with \[ Z_t = \frac{1}{1+\mu_t} \alpha \frac{Y_t}{K_t} \]

\[ \Lambda_{t,+1} = \beta^i C_{t+i}^{-\gamma} / C_t^{-\gamma} \]

- These equations define an "IS curve" that relates spending inversely to the real rate \((1 + r^n_t) \frac{P_t}{P_{t+1}}\) and expectations of the future.

- Interest rates affect investment inversely via \(Q_t\)
Aggregate Supply

output, labor market eq., price index, price setting, capital

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} V_t \]

\[ (1 - \alpha) \frac{Y_t}{L_t} = (1 + \mu_t) a_n \frac{L_t^\varphi}{C_t^{-\gamma}} \]

\[ P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) P_t^{\phi 1-\varepsilon} \right]^{1\over 1-\varepsilon} \]

\[ E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,i} \left[ \frac{P_t^\varphi(f)}{P_{t+i}} \right]^{-\varepsilon} Y_{t+i} \left[ \frac{P_t^\varphi}{P_{t+i}} - \frac{1 + \mu}{1 + \mu_{t+i}} \right] = 0 \]

\[ K_{t+1} = I_t - \frac{1}{2} c \left( \frac{I_t}{K_t} - \delta \right)^2 \cdot K_t + (1 - \delta) K_t \]
Flexible price equilibrium \((Y^*_t, C^*_t, I^*_t, Q^*_t, L^*_t, R^*_{t+1}, K^*_{t+1})\)

→ mark-up \(1 + \mu\) is constant; \(R^*_{t+1} \equiv [(1 + i_t)P_t/P_{t+1}]^*; \Lambda^*_{t,+1} \equiv \beta C^*_{t+1}^{-\gamma}/C_t^{-\gamma}\)

\[
Y^*_t = C^*_t + I^*_t + G
\]

\[
C^*_t = E_t \left\{ (R^*_{t+1}^* \beta)^{-\sigma} C^*_{t+1} \right\}
\]

\[
\frac{I^*_t}{K^*_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q^*_t})
\]

\[
E_t \left\{ \Lambda^*_{t,+1} R^*_{t+1} \right\} = E_t \left\{ \Lambda^*_{t,+1}[\frac{1}{1+\mu} \alpha \frac{Y^*_{t+1}}{K^*_{t+1}} + (1 - \delta)Q^*_{t+1}] / Q^*_t \right\}
\]

\[
Y^*_t = A_t K^*_t^\alpha L_t^{1-\alpha}
\]

\[
(1 - \alpha)\frac{Y^*_t}{L^*_t} = (1 + \mu)a \frac{L^*_t^\varphi}{C^*_{t-\gamma}}
\]

\[
K^*_{t+1} = I^*_t - \frac{1}{2} c\left(\frac{I^*_t}{K^*_t} - \delta\right)^2 \cdot K^*_t + (1 - \delta)K^*_t
\]

RBC with steady state distortion \((\mu > 0)\) and investment adjustment costs.
Loglinear System: Aggregate Demand

Log-linearize around the flexible price steady state with zero inflation:

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} \text{inv}_t \]

\[ c_t = -\sigma [r_t^n - E_t \pi_{t+1} - r] + E_t \{c_{t+1}\} \]

\[ \text{inv}_t - k_t = \frac{1}{\delta c} q_t \]

\[ r_t^n - E_t \pi_{t+1} - r = E_t \{(1 - \nu) z_{t+1} + \nu q_{t+1} - q_t\} \]

with \( z_t = y_t - k_t - \mu_t \);

\( \pi_t = p_t - p_{t-1}, \quad \nu = (1 - \delta)/\left[\frac{\alpha Y}{(1+\mu)K} + 1 - \delta\right] \).
Interest Rates, Asset Prices and Aggregate Demand

Consumption:

\[ c_t = E_t \{ \sum_{i=0}^{\infty} -\sigma (r_{t+i}^n - \pi_{t+1+i} - r) \} \]

Asset price

\[ q_t = E_t \{ (1 - \nu) z_{t+1} - (r_{t+i}^n - \pi_{t+1+i} - r) + \nu q_{t+1} \} \]

\[ = E_t \{ \sum_{i=0}^{\infty} \nu^i [(1 - \nu) z_{t+1} - (r_{t+i}^n - \pi_{t+1+i} - r)] \} \]

Investment

\[ inv_t - k_t = \frac{1}{\delta_c} E_t \{ \sum_{i=0}^{\infty} \nu^i [(1 - \nu) z_{t+1} - (r_{t+i}^n - E_t \pi_{t+1+i} - r)] \} \]

\( c_t \) and \( inv_t \) depend on expected path of \( r_{t+i}^n - E_t \pi_{t+1+i} \)
Log-linear System: Aggregate Supply

output, labor market eq. , NK Phillips Curve and capital

\[ y_t = a_t + \alpha k_t + (1 - \alpha) l_t \]
\[ y_t - l_t = \mu_t + \varphi l_t + \gamma c_t \]
\[ \pi_t = \lambda(-\mu_t) + \beta E_t\{\pi_{t+1}\} \]
\[ k_{t+1} = \delta inv_t + (1 - \delta) k_t \]

\[ \pi_t = p_t - p_{t-1}; \quad \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \]

NK Phillips Curve derived from

\[ p_t = \theta p_t + (1 - \theta)p_t^0 \]
\[ p_t^0 = (1 - \theta\beta)E_t \sum_{i=0}^{\infty}(\theta\beta)^i(\mu_t + p_{t+i}) \].
"Compact" Loglinear System

AD

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} inv_t \]

\[ c_t = E_t \sum_{i=0}^{\infty} -\sigma(r^n_{t+i} - \pi_{t+1+i} - r) \]

\[ inv_t - k_t = \frac{1}{\delta c} E_t \{ \sum_{i=0}^{\infty} \nu^i [(1 - \nu) z_{t+1} - (r^n_{t+i} - E_t \pi_{t+1+i} - r)] \} \]

AS

\[ \pi_t = \lambda(-\mu_t) + \beta E_t \{ \pi_{t+1} \} \]

\[ \mu_t = -\kappa y_t - \gamma c_t + (1 + \kappa)(a_t + \alpha k_t); \quad (\kappa \equiv (1 + \varphi)/(1 - \alpha)) \]

\[ k_{t+1} = \delta inv_t + (1 - \delta) k_t \]

MP

\[ r_t^n = r + \phi_\pi \pi_t + \phi_y (y_t - y_t^*) \]
Model Expressed in Deviations from Flexible Price Equilibrium

\( \tilde{x}_t \equiv x_t - x_t^* \) and since \( k_t \) is predetermined \( \tilde{k}_t = 0 \)

\[
\tilde{y}_t = \frac{C}{Y} \tilde{c}_t + \frac{I}{Y} \tilde{invt}
\]
\[
\tilde{c}_t = E_t \sum_{i=0}^{\infty} -\sigma (r_{t+i}^n - \pi_{t+1+i} - r_{t+1+i}^*)
\]
\[
\tilde{invt} = \frac{1}{\delta c} E_t \left\{ \sum_{i=0}^{\infty} \nu^i [(1 - \nu) \tilde{z}_{t+1} - (r_{t+i}^n - E_t \pi_{t+1+i} - r_{t+1+i}^*)] \right\}
\]
\[
\pi_t = \lambda (-\tilde{\mu}_t) + \beta E_t \{ \pi_{t+1} \}
\]
\[
\tilde{\mu}_t = -\kappa \tilde{y}_t - \gamma \tilde{c}_t
\]
\[
\tilde{k}_{t+1} = \delta \tilde{invt}
\]

\( \tilde{z}_t = \tilde{y}_t - \tilde{\mu}_t; \ r_{t+1}^* \equiv \text{natural flexible price equilibrium real rate.} \)

- Both \( \tilde{c}_t \) and \( \tilde{invt} \), and hence \( \tilde{y}_t \) vary inversely with expected path on interest rate gap \( E\{r_t^n - \pi_{t+1} - r_{t+1}^*\} \)

- \( \tilde{\mu}_t \) varies inversely with \( \tilde{y}_t \) and \( \tilde{c}_t \rightarrow \pi_t \) varies positively with \( \tilde{y}_t \) and \( \tilde{c}_t \).
Taking the Model to Data (August 2007)

  - CEE use methods of moments: match model to IRFs from monetary shock
  - SW estimate full system using Bayesian methods: one shock for each variable.

- Variables the same as in simple baseline presented earlier

- Modifications of structural equations to address following issues
  - Hump-shaped dynamics of output, investment and consumption
  - Smooth behavior of inflation with (relatively) volatile output behavior.
  - The effect of monetary policy on measured productivity.
Key Modifications

1. Habit formation in consumption

\[
E_t \left\{ \sum_{i=0}^{\infty} \beta^i e^{b_{t+i}} \left[ \log (C_{t+s} - hC_{t+s-1}) - \frac{1}{1+\varphi}L_{t+i}^{1+\varphi} \right] \right\}
\]

Induces dependency of \( C_t \) on \( C_{t-1} \) as well as expectations of future, consistent with data.

2. Flow investment adjustment costs

\[
K_{t+1} = (1 - \delta_t)K_t + \zeta_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t
\]

with \( S' > 0 \), \( S'' > 0 \) and \( \zeta_t \equiv \text{investment shocks} \) (literally a shock to the replacement price of capital goods.

Induces dependency of \( I_t \) on \( I_{t-1} \) as well as expectations of future, consistent with data.
Key Modifications (con’t)

3. Variable utilization of capital

\[ Y_t = A_t(U_tK_t)^\alpha L_t^{1-\alpha} \]
\[ \delta_t = \delta(U_t) \]

with \( \delta' > 0, \delta'' > 0 \). MB of increasing utilization: \( \alpha \frac{Y_t}{U_t} \); MC: \( \delta' K_t \).

True TFP = \( A_t \); Measured TFP = \( A_t U_t^\alpha \) → procyclical movements in \( U_t \) can induce procyclical movements in measured TFP

4. Nominal wage rigidity: Households are monopsonistic competitors that supply differentiated labor. Set nominal wages on staggered basis (Calvo style).

\[ \frac{1}{1 + \mu_t} = MC_t = \frac{W_t/P_t}{(1-\alpha)Y_t/L_t} \]

Nominal wage and price rigidity → stickiness in real wages → stickiness in \( MC_t \) and \( \frac{1}{1 + \mu_t} \) → stickiness in inflation.
Post-Mortem

- Model accounts reasonably well for post-war date up to 2007.

- "Investment shock" is most important driving shock
  - Though interpretation of this shock as well as the others is open since they are not directly observed

- However; model breaks down during Great Recession
  - Using data through 2008:Q3 fails to forecast both subsequent downturn and slow recovery
  - Not useful for interpreting unconventional policy interventions

- But not all is lost: Investment shock highly correlated with credit spread, a simple measure of financial stress (Justinano/Primiceri/Tambalotti 2011).
  - Suggests value of more explicit modeling of the shock, allowing for financial frictions.
Figure 1: Model- and VAR-Based Impulse Responses

Legend:
- Solid lines: Benchmark model impulse responses
- Solid lines with +: VAR-based impulse responses
- Grey area: 95% confidence intervals about VAR-based estimates
- Units on horizontal axis: quarters
- * Indicates Period of Policy Shock
- Vertical axis indicate deviations from unshocked path. Inflation, money growth, interest rate: annualized percentage points.
- Other variables: percent.
Figure 1: Year-over-year output growth in the data and in the model with only investment shocks.