Topic 2

The Baseline New Keynesian Model, Monetary Policy, and the Liquidity Trap: Part 2

Mark Gertler NYU

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Outline

Part 1
Household consumption, labor supply, saving decisions, and money demand
Firm labor, capital and price setting decisions
Monetary policy: Taylor rules
Decentralized equilibrium: monetary non-neutrality and inefficient output fluctuations

Part 2
Loglinear model
Aggregate demand and the natural rate of interest
The New Keynesian Phillips curve
Monetary policy design in the basic NK model
The liquidity trap
Loglinearization: Aggregate Demand

Let $x_t = \log X_t - \log X$, except for $r^n_t$, $\pi_t$, $p_t$ and $m_t$ which are in levels

Let $\rho \equiv$ steady state net real interest rate $\beta^{-1} - 1$

Log-linearize around the steady state ($A_t = A$) with zero inflation ($\frac{P_t}{P_{t-1}} = 1$).

$$y_t = c_t$$  \hspace{1cm} (1)

$$c_t = -\sigma [r^n_t - E_t \pi_{t+1} - \rho] + E_t \{c_{t+1}\}$$  \hspace{1cm} (2)

$$r^n_t - E_t \pi_{t+1} - \rho = E_t \{(1 - \nu)(mc_{t+1} + y_{t+1}) + \nu q_{t+1} - q_t\}$$  \hspace{1cm} (3)

where $\pi_t = p_t - p_{t-1}$, $\nu = 1/[\alpha MC_K^Y + 1]$, and $\sigma = \frac{1}{\gamma}$. 

Log-linearization: Aggregate Demand (con’t)

Equation (3) can be rewritten as:

\[ q_t = E_t [(1 - \nu)(m_{c_{t+1}} + y_{t+1}) + \nu q_{t+1} - (r^n_t - E_t \pi_{t+1} - \rho)] \]  (4)

\[ = E_t \sum_{i=0}^{\infty} \nu^i [(1 - \nu)(m_{c_{t+1+i}} + y_{t+1+i}) - (r^n_{t+i} - \pi_{t+1+i} - \rho)] \]  (5)

→ The log price of capital equals the loglinearized expected discounted value of earnings.

* Note that in a model with variable capital, investment will depend positively on \( q_t \).
Log-linearization: Aggregate Supply

\[ y_t = a_t + (1 - \alpha)l_t \] (7)

\[ y_t - l_t = -mc_t + \gamma_n l_t + \gamma c_t \text{ (note: } -mc_t = \mu_t) \] (8)

\[ p_t = \theta p_{t-1} + (1 - \theta)p_t^0 \] (9)

\[ p_t^0 = (1 - \theta \beta)E_t \sum_{i=0}^{\infty} (\theta \beta)^i (mc_{t+i} + p_{t+i}) \] (10)

\[ = (1 - \theta \beta)E_t \sum_{i=0}^{\infty} (\theta \beta)^i (mc_{t+i} + p_{t+i}) \] (11)

Given \( mc_t = \log MC_t - \log MC \rightarrow \)

\[ mc_t + p_t = \log \text{ nominal marginal cost} - \log MC' \]
Log-linearization: Monetary Policy

In the zero inflation steady state $r^n = r = \rho$. (from the consumption euler equation).

Monetary Policy Rule

$$r^n_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - y^*_t) + \nu_t$$ (12)

Money demand

$$m_t - p_t = k + \frac{\gamma}{\gamma_m} y_t - \eta r^n_t$$

with $k = \frac{1}{\gamma_m} \log a_m + \frac{\nu}{\gamma_m} y, \eta = \frac{1}{\gamma_m (R^n-1)}$

Note again: we can ignore money demand since the central bank just adjusts $m_t$ to support its objective for $r^n_t$. 
Log-linearization: Flexible Price Equilibrium

\((y_t^*, c_t^*, l_t^*, r_{t+1}^*)\) determined by

\[y_t^* = c_t^*\]
\[c_t^* = -\sigma \left[ r_{t+1}^* - \rho \right] + E_t\{c_{t+1}^*\}\]
\[y_t^* = a_t + (1 - \alpha)l_t^*\]
\[y_t^* - l_t^* = \gamma_n l_t^* + \gamma c_t^*\]

given \(y_t^* = c_t^* \rightarrow y_t^*, l_t^*\) jointly determined by

\[y_t^* = a_t + (1 - \alpha)l_t^*\]
\[y_t^* - l_t^* = \gamma_n l_t^* + \gamma y_t^*\]

with \(r_{t+1}^*\) given by

\[y_t^* = -\sigma \left[ r_{t+1}^* - \rho \right] + E_t\{y_{t+1}^*\}\]
"IS/AS" Formulation

The above system can be collapsed into two equations: an IS curve that relates output demand inversely to the real interest rate and an aggregate supply curve that relates inflation to excess demand:

\[ IS : y_t = -\sigma(r^n_t - E_t \pi_{t+1} - \rho) + E_t y_{t+1} \]  
\[ AS : \pi_t = \lambda(y_t - y^*_t) + \beta E_t \pi_{t+1} \]

with \( \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \kappa \), and where \( \kappa \equiv \) elasticity of \( mc_t \) w.r.t. \( y_t \)

\[ y^*_t = \frac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t \]

and where the markup (and hence the labor wedge) is countercyclical

\[ mc_t = \kappa(y_t - y^*_t) \rightarrow \mu_t = -\kappa(y_t - y^*_t) \]

\( r^n_t \) is given by the Taylor rule 12.
AS Curve

The Phillips curve (14) is derived from the recursive formulation of equation (10):

\[ p_t^o = (1 - \beta\theta)(mc_t + p_t) + \beta\theta E_t p_{t+1}^o \]  (15)

From the price index equation (9), we get:

\[ p_t - p_{t-1} = \pi_t = \frac{1 - \theta}{\theta}(p_t^o - p_t) \]  (16)

Combining (15) and (16) yields:

\[ p_t^o - p_t = (1 - \beta\theta)mc_t + \beta\theta E_t \left[ p_{t+1}^o - p_{t+1} + p_{t+1} - p_t \right] \]  (17)

\[ \frac{\theta}{1 - \theta}\pi_t = (1 - \beta\theta)mc_t + \beta\theta E_t \left[ \frac{\theta}{1 - \theta}\pi_{t+1} + \pi_{t+1} \right] \]  (18)

\[ \pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}mc_t + \beta E_t \pi_{t+1} \]  (19)
Log-linearization: Connecting $mc_t$ to $y_t - y_t^*$

Log-linearizing equations describing the flexible price equilibrium, we get:

$$y_t^* = a_t + (1 - \alpha)l_t^*$$
$$y_t^* - l_t^* = \varphi l_t^* + \gamma y_t^*$$

which (given $mc = -\mu$) can be combined into

$$y_t^* = \frac{1 + \varphi}{1 + \varphi + (\gamma - 1)(1 - \alpha)}a_t$$ (20)

Similarly, combine (1), (7) and (8) for the sticky price eq.:

$$y_t = \frac{1 + \varphi}{1 + \varphi + (\gamma - 1)(1 - \alpha)}a_t + \frac{mc_t}{(\gamma - 1) + \frac{\varphi + 1}{1 - \alpha}}$$ (21)

Then

$$y_t = y_t^* + \frac{mc_t}{(\gamma - 1) + \frac{\varphi + 1}{1 - \alpha}}$$ (22)
Connecting $mc_t$ to $y_t - y_t^*$ (con’t)

- marginal cost and the output gap.

$$mc_t = \kappa (y_t - y_t^*)$$  \hspace{1cm} (23)

with $\kappa = (\gamma - 1) + \frac{\varphi + 1}{1 - \alpha}$.

- note: $mc_t = -\mu_t$ → countercyclical markup → countercyclical labor wedge

- Combining (19) and (23) yields the New Keynesian Phillips curve (14):

$$\pi_t = \lambda (y_t - y_t^*) + \beta E_t \pi_{t+1}$$  \hspace{1cm} (24)

with $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \kappa$.

Captures short run positive relation between $y_t - y_t^*$ and $\pi_t$.

Forward looking in contrast to traditional PC: $E_t \pi_{t+1}$ enters, not $\pi_{t-1}$.
Baseline New Keynesian Model

Standard representation

\[ IS : \quad y_t = -\sigma(r^n_t - E_t\pi_{t+1} - \rho) + E_t y_{t+1} \]
\[ AS : \quad \pi_t = \lambda(y_t - y^*_t) + \beta E_t \pi_{t+1} \]
\[ MP : \quad r^n_t = \rho + \phi_\pi \pi_t + \phi_\pi (y_t - y^*_t) + \nu_t \]

with

\[ y^*_t = \frac{1+\phi}{1+\phi+(\gamma-1)(1-\alpha)} a_t \]
\[ a_t = \rho a_{t-1} + \varepsilon_{at} \]
\[ \nu_t = \rho_m \nu_{t-1} + \varepsilon_{mt} \]

Monetary policy non-neutral. \( \nu_t \uparrow \rightarrow r^n_t \uparrow \rightarrow y_t \downarrow \rightarrow \pi_t \downarrow \).

Nominal price stickiness key.
CHAPTER 3

an average price duration of four quarters, a value consistent with much of the empirical evidence. As to the interest rate rule coefficients, it is assumed \( a_p = 1.5 \) and \( \phi_0 = 0.5/4 \), in a way consistent with Taylor’s original rule. Finally, \( \phi_1 = 0.5 \), a setting associated with a moderately persistent shock.

Figure 3.1 illustrates the dynamic effects of a contractionary monetary policy shock on a number of macro variables. The shock takes the form of an increase of 25 basis points in \( \pi_t \), which, in the absence of a further change induced by the response of inflation or the output gap, would imply an increase of 100 basis points in the annualized nominal rate, on impact. The responses of inflation and the two interest rates shown in figure 3.1 (and in all subsequent figures) are expressed in annual terms, that is, they are obtained by multiplying by 4 the responses of \( \pi_t \), \( i_t \), and \( r_t \) implied by the quarterly model.

In a way consistent with the analytical results above it is seen that the policy shock generates a decrease in inflation, output (whose response corresponds to that of the output gap, because the natural level of output is not affected by the monetary policy shock), employment, and the real wage. Note also that the increase in the real rate is larger than that of the nominal rate as a result of the decrease in expected inflation. That persistent increase in the real rate is the factor behind the decline of consumption and output, as implied by (25), and given the constancy of the natural rate.

In order to bring about the observed rise in the nominal interest rate, the central bank must engineer a large short-run reduction in the money supply. The calibrated model thus displays a liquidity effect. Note also that the negative response of the price level to the tightening of monetary policy builds up gradually, and is much more muted in the short run than that of the money supply, which instead overshoots its new permanent plateau. In the long run, however, the price level and the money experience a permanent decline of identical size, since real balances revert back to their initial level.

Overall, the dynamic responses to a monetary policy shock shown in figure 3.1 are similar, at least in a qualitative sense, to those estimated using structural vector autoregressive (VAR) methods, as described in chapter 1. Nevertheless, and as emphasized in Christiano, Eichenbaum, and Evans (2005), among others, matching some of the quantitative features of the empirical impulse responses requires that the basic New Keynesian model be enriched in a variety of dimensions.

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\[ \text{See, in particular, the estimates in Galí, Gertler, and López-Salido (2001) and Sbordone (2002), based on aggregate data and the discussion of the micro evidence in chapter 1.} \]

\[ \text{See, e.g., Taylor (1999). Note that empirical interest rate rules are generally estimated using inflation and interest rate data expressed in annual rates. Conversion to quarterly rates requires that the output gap coefficient be divided by 4.} \]
Output Gap and the Natural Rate of Interest

Output gap: $\tilde{y}_t = y_t - y_t^*$; Natural rate of interest $\equiv r_{t+1}^*$

$y_t^*$ and $r_{t+1}^*$ determined in flexible price equilibrium:

$$y_t^* = -\sigma(r_{t+1}^* - \rho) + E_t y_{t+1}^*$$

$$y_t^* = \frac{1+\varphi}{1+\varphi -(\gamma - 1)(1-\alpha)} a_t$$

$\rightarrow$

$$r_{t+1}^* = \rho + \frac{1}{\sigma} \frac{1+\varphi}{1+\varphi -(\gamma - 1)(1-\alpha)} (E_t a_{t+1} - a_t)$$

$$= \rho + \frac{1}{\sigma} \frac{1+\varphi}{1+\varphi -(\gamma - 1)(1-\alpha)} (\rho a - 1) a_t$$

$r_{t+1}^*$ depends on expected productivity growth
The NK Model in Terms of $\tilde{y}_t$ and $\pi_t$

Combining sticky and flex price equilibria $\rightarrow$

$$\tilde{y}_t = -\sigma[(r^n_t - E_t\pi_{t+1}) - r^*_t] + E_t\tilde{y}_{t+1}$$

$$\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}$$

$$r^n_t = \rho + \phi_\pi \pi_t + \phi_\pi \tilde{y}_t + \nu_t$$

with

$$r^*_t = \rho + \frac{1}{\sigma(1+\gamma_n)(1-\gamma)(1-\alpha)}(\rho_a - 1)a_t$$

$\rightarrow \tilde{y}_t$ depends inversely on "interest rate" gap $(r^n_t - E_t\pi_{t+1}) - r^*_t$
The Role of Expectations

We can represent the IS and AS curves as a system of simultaneous first order difference equations in \( \tilde{y}_t \) and \( \pi_t \) conditional on the path of the policy instrument \( r^*_t \).

\[
\tilde{y}_t = -\sigma[(r^n_t - E_t\pi_{t+1}) - r^*_{t+1}] + E_t\tilde{y}_{t+1}
\]

\[
\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}
\]

There are no endogenous predetermined states. Both \( \tilde{y}_t \) and \( \pi_t \) are endogenous at \( t \) and depend on beliefs about the future. \( \rightarrow \) To solve iterate forward

\[
\tilde{y}_t = E_t\sum_i -\sigma[(r^n_{t+i} - E_t\pi_{t+1+i}) - r^*_{t+1+i}]
\]

\[
\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}
\]

\( \tilde{y}_t \) depends inversely on expected path of interest rate gap. (forward guidance matters!)

\( \pi_t \) depends positively on expected path of \( \tilde{y}_t \) (forward looking price setting).
Monetary Policy Design: The "Taylor" Principle

\[ \tilde{y}_t = \sum_i -\sigma[(r^n_{t+i} - E_t\pi_{t+1+i}) - r^*_{t+1+i}] \]

\[ \pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\} \]

\[ r^n_t = \rho + \phi_{\pi} \pi_t + \phi_y \tilde{y}_t + \nu_t \]

Suppose the objective of policy is \( \tilde{y}_t, \pi_t = 0 \).

For a unique solution for \((y_t, \pi_t)\) to exist with \( \lim_{i \to \infty} E_t\{\tilde{y}_{t+i}\} = 0 \) and \( \lim_{i \to \infty} E_t\{\pi_{t+i}\} = 0 \), it must be the case that

\[ \lim_{i \to \infty} E_t\{(r^n_{t+i} - E_t\pi_{t+1+i}) - r^*_{t+1+i}\} = 0. \]

A sufficient condition to ensure convergence is that \( \phi_{\pi} > 1 \).("Taylor" principle": see Gali).
The Taylor Principle and Macroeconomic Stability: Intuition

\[ \tilde{y}_t = \sum_i -\sigma[(r^*_{t+i} - E_t\pi_{t+1+i}) - r^*_{t+1+i}] \]

\[ \pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\} ; \quad r^n_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]

Intuitively, suppose \( r^*_{t+1+i} \uparrow \) (due e.g. to a drop in \( a_t \)) \( \rightarrow \tilde{y}_t \uparrow \) (given \( r^n_t \)) \( \rightarrow \pi_t \uparrow \).

Then \( \phi_\pi > 1 \) \( \rightarrow \) \( r^n_{t+i} \uparrow \) enough to raise real rates \( r^*_{t+i} - E_t\pi_{t+1+i} \rightarrow \)

\( r^n_{t+i} - E_t\pi_{t+1+i} \) converge to \( r^*_{t+1+i} \rightarrow \tilde{y}_{t+i} \) and \( \pi_{t+i} \rightarrow 0 \)

\( \phi_\pi > 1 \) also eliminates self-fulfilling movements in inflation.

Suppose \( E_t\pi_{t+1} \uparrow \rightarrow (r^n_t - E_t\pi_{t+1}) \downarrow \) (given \( r^n_t \)\( \rightarrow \tilde{y}_t \uparrow \)\( \rightarrow \pi_t \uparrow \)

With \( \phi_\pi > 1 \) \( \rightarrow r^n_t \uparrow \) enough to raise real rates, choking off self-fulfilling inflation

Evidence: \( \phi_\pi < 1 \) from mid 60s to late 70s, a period of volatile inflation and output

Conversely, \( \phi_\pi > 1 \) from early 1980s to 2007, the Great Moderation.
Figure 4. The Federal Funds Rate and the Inflation Rate
### TABLE 1
ESTIMATES OF POLICY REACTION FUNCTION

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<th>γ_n</th>
<th>γ_p</th>
<th>ρ</th>
</tr>
</thead>
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<td>Pre-Volcker</td>
<td>0.83</td>
<td>0.27</td>
<td>0.68</td>
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<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.05)</td>
</tr>
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<td>Volcker-Greenspan</td>
<td>2.15</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(0.40)</td>
<td>(0.42)</td>
<td>(0.04)</td>
</tr>
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</table>
The Taylor Principle and Macroeconomic Stability: Formalities

\[ \tilde{y}_t = -\sigma \left[ (r_t^n - E_t\pi_{t+1}) - r_{t+1}^* \right] + E_t \tilde{y}_{t+1} \]
\[ \pi_t = \lambda(\tilde{y}_t) + \beta E_t \pi_{t+1} \]
\[ r_t^n = \rho + \phi_\pi \pi_t + \phi_\gamma \tilde{y}_t + v_t \]

Use the policy rule to eliminate \( r_t^n \) in the IS equation →

\[
\begin{bmatrix}
  y_t \\
  \pi_t
\end{bmatrix}
= A
\begin{bmatrix}
  E_t y_{t+1} \\
  E_t \pi_{t+1}
\end{bmatrix}
+ B \cdot u_t
\]

where \( A \) is 2x2 and \( B \) is 2x1.

Unique solution exists if the two roots of \( A \) lie within the unit circle.

→ unique solution can be obtained through forward iteration.

Sufficient condition for the roots of \( A \) in the unit circle: \( \phi_\pi > 1 \). (Gali p.65)
Optimal Policy Rule: Given objective $\tilde{y}_t, \pi_t = 0$

$$\tilde{y}_t = \sum_i -\sigma[(r^n_{t+i} - E_t\pi_{t+1+i}) - r^*_{t+1+i}]$$

$$\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}$$

Preferable policy rule (ignoring issues of commitment for now):

$$r^n_{t+i} = r^*_{t+1+i} \quad \forall i \geq 0 \rightarrow \tilde{y}_t, \pi_t = 0$$

To ensure $\pi_t \rightarrow 0$, need to specify that policy will adjust if $\pi_t$ deviates from 0:

A rule that accomplishes this is

$$r^n_t = r^*_{t+1} + \phi_{\pi}\pi_t \text{ with } \phi_{\pi} > 1$$

As in the previous case, $\phi_{\pi} > 1$ ensures a determinate solution for $\tilde{y}_t$ and $\pi_t$ (thus ruling out self-fulfilling solutions).

The difference in this case is that $\tilde{y}_t$ and $\pi_t$ go right to 0.
Demand Shocks

Standard approach: preference shifter to induce fluctuations in consumption demand:

Modify utility function as follows:

\[
E_t\left\{ \sum_{i=0}^{\infty} \beta^i e^{b_t+i} \left[ \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right] \right\}
\]

where the preference shock \( b_t \) obeys

\[
b_t = \rho b_{t-1} + \varepsilon_{bt}
\]

→ Consumption euler equation:

\[
e^{b_t} C_t^{-\gamma} = E_t\{\beta e^{b_t+1} C_t^{-\gamma} R_t^n \frac{P_t}{P_{t+1}}\}
\]
Demand Shocks (con’)

In loglinear form (given $\sigma = 1/\gamma$)

\[ c_t = -\sigma[(r^n_t - E_t\pi_{t+1}) - \rho] + E_t\{c_{t+1}\} + \sigma(b_t - E_t\{b_{t+1}\}) \]

\[ = -\sigma[(r^n_t - E_t\pi_{t+1}) - \rho] + E_t\{c_{t+1}\} + \sigma(1 - \rho_b)b_t \]

since $y_t = c_t$:

\[ y_t = -\sigma[(r^n_t - E_t\pi_{t+1}) - \rho] + E_t\{y_{t+1}\} + \sigma(1 - \rho_b)b_t \]

natural rate of output:

\[ y^*_t = -\sigma[r^*_{t+1} - \rho] + E_t\{y^*_{t+1}\} + \sigma(1 - \rho_b)b_t \]

\[ \rightarrow r^*_{t+1} \text{ depends on } b_t \]
IS/AS Model with Demand Shocks

Given $\tilde{y}_t = y_t - y_t^*$

$$\tilde{y}_t = -\sigma[(r^*_t - E_t\pi_{t+1}) - r^*_{t+1}] + E_t\tilde{y}_{t+1}$$

$$\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}$$

with

$$y_t^* = \frac{1 + \varphi}{1 + \varphi + (\gamma - 1)(1 - \alpha)} a_t$$

$$r^*_{t+1} = \rho + \frac{1}{\sigma} \frac{1 + \gamma_n}{1 + \gamma_n - (1 - \gamma)(1 - \alpha)} (\rho a - 1) a_t + (1 - \rho_b) b_t$$

$r^*_{t+1}$ summarizes the effect of $b_t$ and $a_t$ relevant to monetary policy

Optimal to continue to set $r^*_t = r^*_{t+1}$.

Complication: $r^*_{t+1}$ not directly observable (though $\pi_t$ provides information).
3.4.1.2 THE EFFECTS OF A DISCOUNT RATE SHOCK

Figure 3.2 displays the dynamic responses of different macro variables to a discount factor shock, in the form of a decrease in \( z \). Like in the case of the monetary policy shock, the corresponding autoregressive coefficient is set to \( \rho_2 = 0.5 \). The size of the initial shock, \( \phi \), is normalized to \(-0.5\) per annum points, so that \( \phi = (1 - \rho_2) \sigma = \sigma \), implying an increase in the (annualized) natural rate of interest. A decline in \( z \) should be interpreted as a reduction in the weight that households give to current utility relative to future utility. That shift in preferences induces a decline in consumption and hence in aggregate demand.

As shown in Figure 3.2, the decline in \( z \) leads to a contraction in output, employment, inflation, and the real wage. In fact, and given the normalization of the size of the shock, the response of these variables is identical to that describing the effects of a monetary policy tightening, as shown in Figure 3.1. Formally, the reason for this is that \( 1 - \rho_2 \sigma \) and \( \sigma \) enter symmetrically (though with opposite sign) in the system (27) describing the equilibrium dynamics for inflation and the output gap under the interest rate rule considered here. Given that \( z \) does not affect the natural level of output, either the effects on output, employment, and the real wage are also identical.

Discount rate shocks and monetary policy shocks differ in two respects, however: (i) shifts in the discount factor have an effect on the natural rate of interest \( r^*_n \), whereas monetary policy shocks don’t (see 26), and (ii) monetary policy shocks lead to changes in the nominal interest rate, for any given levels of inflation and output, whereas discount factor shocks don’t (see 26). As a result, the effects of the two shocks on the nominal and real interest rates are very different, as a comparison of Figures 3.1 and 3.2 makes clear. In particular, the nominal rate falls in response to a negative discount factor shock, due to the decline in inflation and output. The decline in the nominal rate drags the real rate down, given the sluggish change in expected inflation. The decline in the real interest rate, however, is not sufficient to prevent the overall contraction in economic activity. Note also that the different response of the nominal rate is associated with a very different pattern in the response of the money supply, which now rises in the short run (due to the dominant effect of a lower nominal rate), before it declines and settles at a permanently lower level (the same as the price level, which in turn corresponds to that implied by the monetary policy shock).

3.4.1.3 THE EFFECTS OF A TECHNOLOGY SHOCK

Next, the effects of exogenous changes in \( \alpha_t \) are considered. Given (27) and the analysis of its solution above, it follows that

\[ y_t = \phi \alpha_t \sigma (1 - \rho_z) (1 - \rho_2) \alpha \sigma. \]
Baseline New Keynesian Model: Properties

- $\tilde{y}_t$ depends inversely on current and expected future movements of $(r_{t+i}^n - E_t\pi_{t+1+i})$ relative to $r_{t+1+i}^*$. 

- $\pi_t$ depends positively on current and expected future movements of $\tilde{y}_t$.

- No short run trade-off between $\pi_t$ and $\tilde{y}_t$ for a credible central bank (i.e. a central bank that can commit to keeping $\tilde{y}_{t+i} = 0 \forall i > 0$).
  
  - Requires committing to adjust path of $r_{t+i}^n$ so $(r_{t+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^* = 0 \forall i$.
  
  - Result depends on absence of labor market frictions (otherwise $mc_t$ not simply proportionate to $\tilde{y}_t$).
  
  - If steady state output is inefficiently low, the central might be tempted to inflate.
  
  - If zero lower bound on the nominal rate binds, the economy is susceptible to deflation and output losses.
Liquidity Trap and the Zero Lower Bound (ZLB)

- Liquidity trap: a situation where the central bank cannot stimulate the economy by reducing the short term interest rate.

- Emerges when ZLB constraint on net nominal interest rate binds
  
  \[ R^n_t - 1 \geq 0 \iff R^n_t \geq 1 \iff \log R^n_t = r^n_t \geq 0 \]
  
  - From earlier: desirable to set \( r^n_t = r^*_t+1 \) (natural interest rate)

- ZLB binds if natural real rate \( R^*_t+1 < 1 \iff r^*_t+1 < 0 \) where \( r^*_t+1 = \log R^*_t+1 \)

- Deflationary spiral can emerge, with \( \tilde{y}_t < 0 \) and \( \pi_t < 0 \).
Liquidity Trap and the Zero Lower Bound (con’t)

• Suppose:
  – for \( k \) periods \( r^*_{t+1+i} < 0 \)
  – central bank pushes \( r^*_{n_{t+i}} \) to ZLB over this period \( \rightarrow r^*_{n_{t+i}} = 0 \)

\[
\tilde{y}_t = E_t\{ \sum_{i=0}^{k-1} -\sigma[(-E_t\pi_{t+1+i})-r^*_{t+1+i}]+\sum_{i=k}^{\infty} -\sigma[(r^*_{n_{t+i}}-E_t\pi_{t+1+i})-r^*_{t+1+i}] \}
\]

• If for \( i \geq k + 1, (r^*_{n_{t+i}} - E_t\pi_{t+1+i}) = r^*_{t+1+i} \):

\[
\tilde{y}_t = E_t\{ \sum_{i=0}^{k-1} -\sigma[(-E_t\pi_{t+1+i})-r^*_{t+1+i}] \}
\]

• \( r^*_{t+1+i} < 0 \) → a liquidity trap emerges with \( \tilde{y}_{t+i}, \pi_{t+i} < 0 {\ }\text{until} {\ } i \geq k + 1. \)
Escaping A Liquidity Trap

- Way out - commit to inflation after $r_{t+1+i}^*$ becomes positive.

$$\tilde{y}_t = \sum_{i=0}^{k-1} -\sigma [(-E_t\pi_{t+1+i}) - r_{t+1+i}^*] + \sum_{i=k}^{\infty} -\sigma [(r_{t+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^*]$$

- That is commit to $[(r_{t+1+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^*] < 0$ for $i \geq k + 1$.

- Note that this implies $\pi_{t+i} > 0$ if this commitment is kept $\Rightarrow$ credibility problem: Incentive to renege when out of liquidity trap.

- Fiscal policy may be an alternative (to raise $r_{t+1+i}^*$)

- In an economy with financial market frictions, credit policy may also be an alternative.