Homework 2

EL-GY 9123 – Game Theory for Multi-Agent Systems

Fall 2014, New York University

Due Date: Oct. 10, 2014

Reading Assignment 1. Read [BO] Chapters 2 and 3.

Reading Assignment 2. Read [FT] Chapters 3 and 4.

Reading Assignment 3. Read the following papers:

- Reinhard Selten, "Multistage Game Models and Delay Supergames," Nobel Seminar, December 9, 1994
- Reinhard Selten, "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," International Journal of Game Theory, 4(1):25–55, 1975.
- Roger B. Myerson, "Refinements of the Nash Equilibrium Concept," International Journal of Game Theory, 7(2):73–80, 1978.
- Monderer, D., & Shapley, L. S. (1996), "Potential games," Games and economic behavior, 14(1), 124-143.
- Dubey, Pradeep. "Inefficiency of Nash equilibria." Mathematics of Operations Research 11, no. 1 (1986): 1-8.
- Leo K. Simon and Maxwell B. Stinchcombe, "Equilibrium Refinement for Infinite Normal-Form Games," Econometrica, 63(6):1421-1443, November 1995
- Sergiu Hart & Andreu Mas-Colell, "A Simple Adaptive Procedure Leading to Correlated Equilibrium," Econometrica, Econometric Society, Econometric Society, vol. 68(5), pages 1127-1150, 2000.

Problem 1. Problem 11 from [BO] page 71.

Problem 2. Problems 12 and 13 from [BO], page 71.

Problem 3. Problems 18 and 19 from [BO], page 73.

Problem 4. Problems 12 rom [BO], page 155.

Problem 5. Consider the bi-matrix game

$$P1 : \begin{bmatrix} 0 & 5 \\ 1 & 4 \end{bmatrix}, \quad P2 : \begin{bmatrix} 4 & 5 \\ 1 & 0 \end{bmatrix},$$

where both players are minimizers. Payer 1's choices are denoted by U (up) and D (down). Player 2's choices are denoted by L (left) and R (right). This game admits two pure-strategy NE, (row 1, column 1), and (row 2, column 2), with cost levels of $(0, 4)$ and $(4, 0)$, respectively. It also admits a mixed-strategy NE, where players choose their two alternatives each with equal probabilities 0.5 and 0.5. We now wish to obtain a correlated equilibrium for this game. Consider a signal device that has 3 equally likely states: A, B, and C. Player 1 observes perfectly the state A (but cannot differentiate between A and B). The players choose their actions based on the signal that they observe.

(i) **Formulate** the extensive form of this game with the random device (the tree structure), and obtain its normal form. Show that the two pure-strategy NE above are also NE of this normal form (and thus of the extensive form). **Further show** that the following strategy pair for the players is also a NE of the game with the signaling device: Player 1 picks row 1 if he sees A, and picks row 2 otherwise; Player 2 picks column 2 if he sees C, and picks column 1 otherwise. **Show** also that this choice leads to an expected cost pair for the players that is uniformly better than any convex combination of the two pure-strategy NE without the signaling device.

(ii) **Show** that the signaling device above leads to a correlated equilibrium, where players pick the entries (0,4), (1, 1) and (4,0), with equal probability 1/3. [You have to prove that this is indeed a correlated equilibrium, satisfying the conditions derived in class.]

Problem 6. Exercise 2.8 from [FT], page 63. (Only need to solve for Fig. 2.5)

After solving Problems 1-6 by hand, you can use the following software tools to verify your results:

- ComLab Games: `http://www.comlabgames.com/efg/`
- Gambit: `http://www.gambit-project.org`