Homework 3

EL-GY 9123 – Game Theory for Multi-Agent Systems

Fall 2014, New York University

Due Date: Oct. 22, 2014

Announcements

- When you hand in your homework, please place it in the envelop on the door of my office LC 200A.

Reading Assignment 1. Read [BO] Chapter 4.1-4.4, 4.6-4.7.

Reading Assignment 2. Read [FT] Chapter 4.1-4.4, 4.7. Read Chapter 2 of [RB].

Reading Assignment 3. For a quick tutorial on optimization, read `https://www.economics.utoronto.ca/osborne/MathTutorial/IND.HTM`

Reading Assignment 4. For a review of results related to vectors and matrices, consult the following link `http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/3274/pdf/imm3274.pdf`

You can use MATLAB, MATHEMATICA or other computational tools (e.g. those listed at `http://en.wikipedia.org/wiki/List_of_optimization_software`) to compute optimal solutions where needed.

Reading Assignment 5. Read the following papers:

- J.B.Rosen, "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," Econometrica, 33(3):520-534, July 1965
- Shu Li and Tamer Başar, "Distributed Algorithms for the Computation of Noncooperative Equilibria," Automatica, 23(4):523-533, 1987
- Roy Radner, "Team Decision Problems," Annals of Mathematical Statistics, 33(3):857-881, September 1962
- Tamer Başar, "An Equilibrium Theory for Multiperson Decision Making with Multiple Probabilistic Models," IEEE Transactions on Automatic Control, 30(2):118-132, 1985

- Shu Li and Tamer Başar, "Asymptotic Agreement and Convergence of Asynchronous Stochastic Algorithms," IEEE Transactions on Automatic Control, 32(7):612-618, 1987

Problem 1. Consider the following version of prisoner's dilemma:

	D	C
D	(1,1)	(5,0)
C	(0,5)	(4,4)

,

where $P1$ is the row player while $P2$ is the column player. Both players are maximizers. Suppose this game is played repeatedly with a payoff discount factor of α. For what values of α is the trigger strategy (discussed in class) a subgame-perfect equilibrium?

Problem 2. Exercise 4.4 from [FT], page 139 (The Rubinstein-Stahl problem is explained in Section 4.4 of the book. You have to just read Section 4.4.1 to understand this problem.)

Problem 3. Problem 3 from [BO], pp. 205-206.

Problem 4. You are given the static zero-sum game with cost function

$$J(u,w) = |Ax + Bu + Dw|_Q^2 + |u|^2 - \gamma^2 |w|^2,$$

where $x \in \mathbb{R}^n$ is a given vector, $u \in \mathbb{R}^{r_1}$ and $w \in \mathbb{R}^{r_2}$ are controlled by Players 1 and 2, respectively, A, B, D are matrices of compatible dimensions, Q is a nonnegative definite matrix of dimensions $n \times n$, γ is a positive scalar parameter, and $|\cdot|$ denotes an appropriate Euclidean (semi-) norm, with subscript standing for the weight, that is $|x|_Q^2 = x^T Q x$. Player 1 is the minimizer, and Player 2 is the maximizer.

(i) Write down the precise conditions under which the game admits a pure-strategy saddle point, and also obtain the saddle-point solution.

(ii) Consider the following computational scheme:

$$u(k+1) = \arg\min_u J(u, w^{(k)}), k = 0, 1, ...$$

$$w(k+1) = \arg\min_w J(u^{(k)}, w), k = 0, 1, ...$$

which we start with $u^{(0)} = 0, w^{(0)} = 0$. Assume that the two sequences thus generated converge, say to

$$\lim_{k\to\infty} u^{(k)} = \bar{u}, \quad \lim_{k\to\infty} w^{(k)} = \bar{w}.$$

Is the pair $(\bar{u}, \bar{w})$ necessarily a saddle-point solution?

(iii) Under what conditions on $A, B, D, Q,$ and γ do the sequences generated by the algorithm above converge (or have converging subsequences)?

Problem 5. Problem 9 from [BO], page 208.

Problem 6. Problem 13 from [BO], pages 209-210.

Problem 7. Problem 5 from [BO], pages 206-207. (This is a bonus problem. You will get extra credits.)