Homework 4

EL-GY 9123 – Game Theory for Multi-Agent Systems

Fall 2014, New York University

Due Date: Nov. 21, 2014

Reading Assignment 1. Read [BO] Chapter 4.

Reading Assignment 2. Read [FT] Chapters 6,7.

Reading Assignment 3. Read [RB] Chapter 3.

Reading Assignment 4. Please read the following materials:

- J. C. Harsanyi, 1967/1968, "Games with Incomplete Information Played by Bayesian Players, I-III." *Management Science*, 14 (3): 159-183 (Part I), 14 (5): 320-334 (Part II), 14 (7): 486-502 (Part III).
- I-K. Cho and D. M. Kreps, "Signaling games and stable equilibria," *Quarterly Journal of Economics*, 102:179-221, 1987.
- J. Jee, A. Sundstrom, S. E. Massey, and B. Mishra, "What can information-asymmetric games tell us about the context of Crick's 'Frozen Accident'?" Journal of the Royal Society Interface 10 (88), 2013.
- V. P. Crawford and J. Sobel, "Strategic Information Transmission," Econometrica, Econometric Society, vol. 50(6), 1982, pages 1431-51, November.
- E. Maskin, Nobel Prize Lecture: `http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2007/maskin-lecture.html`
- R. Myerson, Nobel Prize Lecture: `http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2007/myerson-lecture.html`

Problem 1. Consider the two-player nonzero-sum stochastic game with cost functions

$$L_i(u_1, u_2; \xi_i, \eta_i) = \frac{1}{2}(u_i)^2 + \eta_i u_i u_j + \xi_i u_i, i, j = 1, 2, i \neq j,$$

where $\xi_i, \eta_i, i = 1, 2$, are independent random variables taking values in the interval $(1, 2)$. We are interested in the Nash equilibrium of this game under two different types of information available to the players.

(i) Suppose that the players know the precise values of the 4 random variables, say $\xi_i = a_i, \eta_i = b_i, i = 1, 2$, and this is common information. For what values of the 4 parameters $a_i, b_i, i = 1, 2$, does the game admit a Nash equilibrium in pure strategies? Find the corresponding Nash equilibrium. Does the game admit any proper (that is, inner) mixed-strategy Nash equilibrium? Compute the expected values of L_1 and L_2 under the pure-strategy Nash equilibrium you have obtained.

(ii) Is the game in (i) above strategically equivalent to a team problem? If it is, for what values of $a_i, b_i, i = 1, 2$, does the team admit a unique team-optimal solution?

(iii) Now assume that both players know that the random variables are all uniformly distributed over the interval $(1,2)$. Player 1 knows the precise values of ξ_1 and η_1, but does not know the values of ξ_2 and η_2 (but knows that each one is uniformly distributed on $(1,2)$). Symmetrically, Player 2 knows the precise values of ξ_2 and η_2, but does not know the values of ξ_1 and η_1 (but knows that each one is uniformly distributed on $(1,2)$). Obtain the Nash equilibrium of this stochastic game, and discuss whether it exists for all observed values of ξ_1 and η_1 (by Player 1) and ξ_2 and η_2 (by Player 2). Compute the expected values of L_1 and L_2 under the Nash equilibrium you have obtained. How do these values compare with those obtained in (i) ?

(iv) Is the game in (iii) above strategically equivalent to a team problem? If it is, obtain the team-optimal solution.

Problem 2. Consider the game of Problem 1 above, with the only modification being that η_2 takes values in $(-2,-1)$.

(i) The players are now assumed to know the precise values of η_1 and η_2 to be $\eta_1 = \frac{3}{2}, \eta_2 = -\frac{3}{2}$, and this is common information to both. They also know that ξ_1 and ξ_2 are independent identically distributed zero-mean unit variance Gaussian random variables defined on $(-\infty,+\infty)$, and this is common information to both players. Further Player i observes the value of ξ_i perturbed additively by independent identically distributed zero-mean unit variance Gaussian noise v_i, this being so for each $i=1,2$; that is Player $i (i=1,2)$ observes $y_i = \xi_i + v_i$, and determines her action based on this measurement, namely $u_i = \gamma_i(y_i)$, and this statistical information is common information to both players. Obtain the Nash equilibrium of this stochastic game.

(ii) How would the solution above change if Player 1 also has access to y_2 (that is, $u_1 = \gamma_1(y_1, y_2)$), but Player 2 still has access to only y_2. Obtain complete expressions for the Nash equilibrium.

Problem 3. Consider a 3-player deterministic quadratic nonzero-sum game, where the action variable of Player i is $u_i, i = 1,2,3$, taking values of the real line. The cost functions of the players are given by

$$L_1(u_1,u_3) = \frac{1}{2}(u_1)^2 - a_1 u_1 u_3 - b_1 u_1,$$

$$L_2(u_2,u_1) = \frac{1}{2}(u_2)^2 - a_2 u_1 u_2 - b_2 u_2,$$

$$L_3(u_3,u_2) = \frac{1}{2}(u_3)^2 - a_3 u_3 u_2 - b_3 u_3,$$

for Players 1-3, where a_is and b_i's are known parameters.

(i) For what values of the six parameters that define the game, there exists a unique Nash equilibrium?

(ii) Consider the 3-player game with a hierarchical decision structure as below. In each of the two scenarios below, obtain the equilibrium solution (as specified) whenever it exists, and obtain necessary and sufficient conditions for existence and uniqueness.

- Player 1 is leader and Players 2 and 3 are followers, with Nash equilibrium solution among the two followers and Stackelberg solution between the leader and the followers group.
- A 3-level hierarchy, with Player 1 as leader at the top, Player 2 as the intermediate player (first follower), and Player 3 as the second follower. Solution concept is Stackelberg.

Problem 4. Exercise 6.3 from [FT].

Problem 5. Exercise 6.4(a) from [FT].

Problem 6. Exercise 6.4(b) from [FT]. Note: A risk-averse bidder is one whose goal is to maximize a concave function of his/her own payoff, rather than the payoff directly. This function is called the utility in this problem. We considered risk-neutral bidders in class; risk-neutral bidders maximize their payoffs. In other words, the utility of risk-neutral bidders is simply their payoffs. (Read and understand Example 6.6 before you do this problem and the next two problems. Also check out `http://en.wikipedia.org/wiki/Risk_aversion`)

Problem 7. Exercise 6.5(a) from [FT].

Problem 8. Exercise 6.5(b) from [FT].