Homework 5

EL-GY 9123 – Game Theory for Multi-Agent Systems

Fall 2014, New York University

Due Date: Dec. 12, 2014

Reading Assignment 1. Read [GO]: Chps IX, X (pp. 211-224), XI, XII (pp. 261-303), and XIII (pp. 313-334).

Reading Assignment 2. Read [FT]: Chapter 8.

Reading Assignment 3. Read the following materials:

1. L. Shapley's Nobel Prize Lecture
   `http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2012/shapley-lecture.html`

2. L.S. Shapley, "Cores of convex games," Intl. J. Game Theory, 1:11-26, 1971.

3. J. Nash, "Two-person cooperative games." Econometrica: Journal of the Econometric Society (1953): 128-140.

4. Owen, Guillermo. "Multilinear extensions of games." Management Science 18, no. 5-part-2 (1972): 64-79.

5. Singh, C., Saswati S., Alireza A., and Anurag K., "Cooperative profit sharing in coalition-based resource allocation in wireless networks." IEEE/ACM Transactions on Networking (TON) 20, no. 1 (2012): 69-83.

6. N. Semret, RR-F. Liao, A. T. Campbell, and A. A. Lazar. "Pricing, provisioning and peering: dynamic markets for differentiated Internet services and implications for network interconnections." Selected Areas in Communications, IEEE Journal on 18, no. 12 (2000): 2499-2513.

7. Yaiche, H, R. R. Mazumdar, and C. Rosenberg. "A game theoretic framework for bandwidth allocation and pricing in broadband networks." IEEE/ACM Transactions on Networking (TON), 8, no. 5 (2000): 667-678.

8. Liu, Yu, Cristina Comaniciu, and Hong Man. "A Bayesian game approach for intrusion detection in wireless ad hoc networks," In Proceeding from the 2006 workshop on Game theory for communications and networks, p. 4. ACM, 2006.

9. D. Monderer, L. S. Shapley, "Potential Games," Games and Economic Behavior, Volume 14, Issue 1, May 1996, Pages 124-143.

10. D. M. Topkis, "Equilibrium points in nonzero-sum n-person sub-modular games." SIAM Journal on Control and Optimization 17.6 (1979): 773-787.

11. H. Shen and T. Başar, “Pricing under information asymmetry for a large population of users,” Telecommunication Systems, 2010.

Problem 1. Consider a third-price auction, i.e., an object is given to the highest bidder, but the bidder pays an amount equal to the third-highest bid. Assume that the valuations of the N bidders are i.i.d. Let f_Θ and $F_\Theta(\theta)$ be the pdf and cdf, respectively, of each bidders valuation. Assume that $\ln F_\Theta$ is a concave function. Show that the bidding strategy

$$\mu(\theta_i) = \theta_i + \frac{F_\Theta(\theta_i)}{(N-2)f_\Theta(\theta_i)}, \forall i.$$

is a symmetric BNE for this problem. Note: Assume differentiability and sufficiency of the first-order necessary conditions as and when required.

Problem 2. Suppose that there is a single buyer and a single seller for some object. The buyer has a valuation θ which is her private information. The seller knows that that θ is drawn from a pdf f_Θ. Suppose that the seller sets a price p at which he is willing to sell. Show that the optimal p is $\Psi^{-1}(0)$, where Ψ is the virtual valuation in Myersons mechanism. Note: You can just use the first-order necessary condition only to determine the optimal solution.

Problem 3. Exercise 4.5 of [FT].

Problem 4. Consider a two-person game with (u, v) denoting the utility levels of the two players. Assume that this pair is restricted to an ellipsoidal region defined by $(u-1)^2 + 4v^2 \leq 4$. Find the Nash bargaining solution to this game when

(i) The conflict payoff is (0, 0).

(ii) The conflict payoff is (1, 0).

Problem 5. Read section X.5 of Owen, and solve Problem 4 on page 233.

Problem 6. Consider the 3-person cooperative game with the characteristic function: $v(\emptyset) = v(\{2,3\}) = v(\{1,3\}) = v(\{1,2\}) = 0, v(\{1\}) = v(\{2\}) = v(\{3\}) = 2, v(\{1,2,3\}) = 3$

(i) Is it a constant-sum game?

(ii) Is it a superadditive game?

(iii) Show that the core of this game, $C(v)$, is empty.

Problem 7. Show that the 3-person game with the characteristic function $v(\emptyset) = v(\{2,3\}) = v(\{1,3\}) = v(\{1,2\}) = 0, v(\{1\}) = v(\{2\}) = v(\{3\}) = 1, v(\{1,2,3\}) = 3$ has a nonempty core, and obtain it.

Problem 8. Problem 1 from the text Owen, page 311.

Problem 9. Evaluate the Shapley vector for the four-person game defined by the characteristic function:

$$v(\emptyset) = 0, v(\{i\}) = 0, i = 1,2,3,4, v(\{2,3\}) = v(\{3,4\}) = v(\{2,4\}) = 0, v(\{1,2,3,4\}) = 1$$

and $v(\{i,j\}) = v(\{i,j,k\}) = 1$ for all other two- or three-person coalitions.

Problem 10. Read the following paper

- X. Shen and L. Deng, "Game theory approach to discrete H_∞ filter design," Signal Processing, IEEE Transactions on , vol.45, no.4, pp.1092,1095, Apr 1997.

and find the H_∞ filter for the following system

$$x_{k+1} = \begin{pmatrix} 0.5079 & 0.7594 \\ -0.7594 & 0.2801 \end{pmatrix} x_k + \begin{pmatrix} 0.4921 \\ 0.7594 \end{pmatrix} w_k,$$

$$y_k = (0 \;\; 1)x_k + v_k,$$

$$z_k = (1 \;\; 0)x_k,$$

where $x_k \in \mathbb{R}^2, z_k \in \mathbb{R}^2, z_k, y_k \in \mathbb{R}$, and $v_k, w_k \in \mathbb{R}$ are process and measurement noise respectively. The initial condition is x_0.

- State the H_∞ filter to obtain the state estimate $\hat{x}_k$ and the measurement estimate $\hat{z}_k$.
- State the Kalman filter (i.e., $\gamma_K = 0$) to obtain the state estimate $\hat{x}_k$ and the measurement estimate $\hat{z}_k$.
- Find the gain of the Kalman filter and H_∞ filter with $\gamma_H = 1.24$.
- Use MATLAB simulations to compare the performance of two filters.