Midterm Exam

EL-GY 9213 – Game Theory for Multi-Agent Systems

Fall 2014, New York University

1:00pm – 3:30pm, October 24, 2014

- The exam consists of 7 problems, and there are 100 pts in total.
- You will be allowed to bring 2 8.5x11 sheets (4 pages) of handwritten notes.
- You do not need calculators for the exam.

Problem 1 (10pt)**.** State whether the following statements are true or false. Justify your answer.

(a) For a finite 2-person nonzero-sum matrix game, there always exists a mixed strategy Nash equilibrium, a correlated equilibrium, and a mixed strategy Stackelberg equilibrium.

(b) Consider the following two games with two payoff players (P1 is a row player, and P2 is a column player): $\mathcal{G}_1$ is a nonzero-sum game (Both players are maximizers), whereas $\mathcal{G}_2$ is a zero-sum game (P1 is a maximizer and P2 is a minimizer).

$\mathcal{G}_1$:

	b_1	b_2
a_1	(7,-10)	(-7,4)
a_2	(1,-4)	(5,-8)

$\mathcal{G}_2$:

	b_1	b_2
a_1	7	-7
a_2	1	5

$\mathcal{G}_1$ and $\mathcal{G}_2$ are strategically equivalent.

Problem 2 (10pt)**.** Find all the Nash equilibria (pure or mixed) of the following matrix game. P1 is a row player, and P2 is a column player. Both players are maximizers. Hint: Use strict dominance to eliminate rows or columns.

	b_1	b_2	b_3	b_4
a_1	(-2,2)	(0,2)	(11,-3)	(5,-6)
a_2	(0,0)	(1,3)	(7,0)	(4,-3)
a_3	(-5,3)	(-3,2)	(4,2)	(-2,1)
a_4	(-6,2)	(-4,1)	(3,1)	(-4,3)

Problem 3 (20pt)**.** Consider the following two-person non-cooperative game with players P1 and P2 (P1 is a row player, and P2 is a column player). Both players are maximizers.

	L	R
U	(5,1)	(0,0)
D	(4,4)	(1,5)

(a) Find all pure and mixed strategy Nash equilibria of this game.

(b) Define the following joint probabilities:

$$\begin{aligned} p_{UL} &= \mathbb{P}(\text{P1 plays U, P2 plays L}), \\ p_{UR} &= \mathbb{P}(\text{P1 plays U, P2 plays R}), \\ p_{DL} &= \mathbb{P}(\text{P1 plays D, P2 plays L}), \\ p_{DR} &= \mathbb{P}(\text{P1 plays D, P2 plays R}), \end{aligned}$$

and $p_{UL}+p_{UR}+p_{DL}+p_{DR} = 1$. Write down all the constraints that these probabilities must satisfy to be a correlated equilibrium that gives a higher payoff to both players than the mixed strategy NE(s) obtained in (a).

(c) Find a correlated equilibrium that satisfies the constraints.

Problem 4 (10pt)**.** Consider the following nonzero-sum game where the entries of the matrix denotes the players costs for taking certain actions. Thus, each players goal is to minimize its cost (instead of maximizing its payoff). We will call the row player P1 and the column player P2. The available actions for P1 are $\{U, M, D\}$, and the available actions for P2 are $\{L, M, R\}$.

	L	M	R
U	(0,-1)	(2,1)	(3/2, -2/3)
M	(1,2)	(1,0)	(3,1)
D	(-1,0)	(2,1)	(2, -1/2)

(a) Find a Stackelberg equilibrium with P1 as the leader

(b) Find a Stackelberg equilibrium with P2 as the leader.

Problem 5 (15pt)**.** Players 1 and 2 must decide whether or not to carry an umbrella when leaving home. They know that there is a 50-50 chance of rain. Each player's payoff is

- -5 if he doesn't carry an umbrella and it rains,
- -2 if he carries an umbrella and it rains,
- -1 if he carries an umbrella and it is sunny,
- 1 if he doesn't carry an umbrella and it is sunny.

Player 1 learns the weather before leaving home; Player 2 does not, but he can observe Player 1's action before choosing his own. Formulate the extensive form of game. Specify the information sets of the players. You do not need to solve the game.

Problem 6 (15pt)**.** In this problem, we consider extensive-form zero-sum games, in which P1 acts first, followed by P2.

(a) Write down the normal form representation (matrix form) of the following game described by Fig. 1. You do not have to solve this game. Note: Recall that a dotted line represents the fact that a player does not know which of the nodes it is in. The payoff to P1 is given in the leaf nodes.

(b) Write down the normal form representation of the following game described by Fig. 2. Find all the pure-strategy saddle points of the normal form game. Among them, identify the one which is subgame-perfect for the extensive-form game.

Problem 7 (20pt)**.** Consider the static zero-sum game with cost function

$$J(u, w) = (x + u + w)^2 + u^2 - \gamma^2 w^2,$$

where $x \in \mathbb{R}$, $u \in \mathbb{R}$ and $w \in \mathbb{R}$ are controlled by Players 1 and 2, respectively. Player 1 is the minimizer, and Player 2 is the maximizer. γ is a positive parameter.

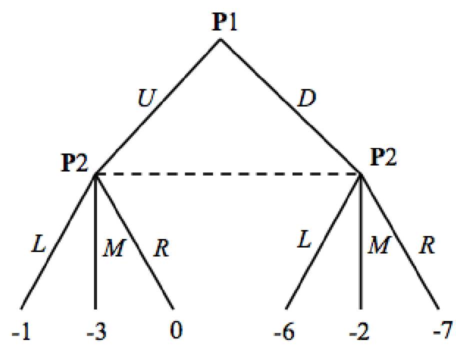

Figure 1: Problem 6 (a) Extensive-form game of a two-person zero-sum game.

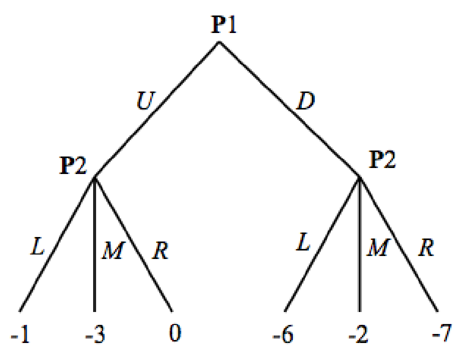

Figure 2: Problem 6 (b) Extensive-form game of a two-person zero-sum game.

(i) Under what values of γ, the game has a saddle point?

(ii) Consider the following computational scheme:

$$u(k+1) = \arg\min_{u} J(u, w^{(k)}), k = 0, 1, ...$$

$$w(k+1) = \arg\min_{w} J(u^{(k)}, w), k = 0, 1, ...$$

which we start with $u^{(0)} = 0, w^{(0)} = 0$.

Under what values of γ, the above algorithm converges?