Active static and seismic earth pressure for c–φ soils

Magued Iskander, PhD, PE, F.ASCE
Professor & Head,
Civil & Urban Engineering Department
• Methods based on Mononobe-Okabe method:
  • Require prior knowledge of failure surface
  • Require assumption of lateral thrust inclination
  • Difficult to apply in layered soils and presence of water table
  • Stress based solutions are scarce and apply to limited cases
  • Develop general Rankine (Stress-based) solution
• Rigid retaining wall
• Frictional-cohesive soil
• Horizontal & vertical acceleration
• Inclined backfill
• Inclined wall
“If the stress on a given plane in a body be in a given direction, the stress on any plane parallel to that direction must be in a direction parallel to the first mentioned plane.”

Rankine (1858)
• Modify acceleration field

• Modify density

• Conjugate element

\[ g_\theta = \sqrt{(g \pm a_v)^2 + a_h^2} \]

\[ \theta = \tan^{-1} \left( \frac{a_h}{g \pm a_v} \right) = \tan^{-1} \left( \frac{k_h}{1 \pm k_v} \right) \]

\[ \gamma_\theta = \frac{\gamma}{\cos \theta} (1 \pm k_v) \]
Mohr circle used to derive stresses

- Conjugate element used to derive stress on a plane parallel to the wall
Stress acting on wall

\[ \sigma'_a \cos \alpha \frac{\cos (\beta + \theta)}{\cos (\beta - \omega)} = \sigma'_\theta \frac{\sin^2 (\theta + \omega)}{\cos (\beta - \omega)} + \sigma'_\beta \cos (\beta - \omega) \]

\[ \sigma'_a = \gamma z K_a = \gamma z \left( \frac{\cos (1 \pm k_v)(\sin^2 (\theta + \omega) - \cos^2 (\beta - \omega))}{\cos \alpha \cos (\beta + \theta) \cos (\theta)} + \frac{2(J_a/\gamma z) \cos^2 (\beta - \omega)}{\cos \alpha} \right) \]

\[ K_a = \frac{\cos (1 \pm k_v)(\sin^2 (\theta + \omega) - \cos^2 (\beta - \omega))}{\cos \alpha \cos (\beta + \theta) \cos (\theta)} + \frac{2(J_a/\gamma z) \cos^2 (\beta - \omega)}{\cos \alpha} \]

\[ J_a = \frac{1}{\cos^2 \phi} \left( \frac{\gamma z \cos \beta \cos (\beta + \theta)(1 \pm k_v)}{\cos \theta} + c \cos \phi \sin \phi \right) \]

\[ - \sqrt{\gamma^2 z^2 \cos^2 \beta (1 \pm k_v)^2 \left( \frac{\cos^2 (\beta + \theta) - \cos^2 \phi}{\cos^2 \theta} \right) + c^2 \cos^2 \phi + \frac{2c \gamma z \cos \phi \sin \phi \cos \beta \cos (\beta + \theta)(1 \pm k_v)}{\cos \theta}} \]
• One of the features of the method is that the obliquity angle on the wall can be analytically obtained:

\[
\alpha_a = \tan^{-1} \left( \frac{2 \cos \theta \cos (\beta + \theta) J_a \cos (1 \pm k_y) \gamma \nearrow - 1}{2 \left( \frac{2 \cos \theta \cos (\beta + \theta) J_a \cos (1 \pm k_y) \gamma \nearrow - 1}{\gamma \nearrow} \right) \sin 2(\beta - \omega) + \sin 2(\theta + \omega) \right)
\]

• Horizontal stress on the wall:

\[
\sigma_{AEH}' = \sigma_a' \cos (\alpha_a + \omega)
\]
Importance of Obliquity
(for vertical wall and horizontal backfill)

Variation of $\alpha$ with horizontal acceleration for different values of $\phi$

Variation of $\alpha$ as a function of $\phi$ for vertical walls, with horizontal backfill
### Example – Layered Backfill

![Diagram of Layered Backfill](image)

**Layer 1**
- $\gamma = 23$ kN/m³
- $\phi = 30^\circ$
- $c = 0$

**Layer 2**
- $\gamma = 20$ kN/m³
- $\phi = 40^\circ$
- $c = 0$

**Depth Along Wall Length**

<table>
<thead>
<tr>
<th>$Z_w$ (m)</th>
<th>Depth Along Length of Wall, $Z_i$ (m)</th>
<th>$Z$ (m)</th>
<th>$J_s$ (Eq. 13)</th>
<th>$\alpha_o$ (°) (Eq. 18)</th>
<th>$K_o$ (Eq. 16)</th>
<th>$\sigma_o'$ (kPa) (Eq. 15)</th>
<th>$\sigma_{AEH}^*$ (kPa) (Eq. 19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.75</td>
<td>29.7</td>
<td>0.698</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6.00 (Layer 1)</td>
<td>6.21</td>
<td>6.28</td>
<td>0.75</td>
<td>29.7</td>
<td>0.698</td>
<td>78.98</td>
<td>56.14</td>
</tr>
<tr>
<td>6.00 (Layer 2)</td>
<td>6.21</td>
<td>6.28</td>
<td>0.69</td>
<td>39.56</td>
<td>0.567</td>
<td>64.09</td>
<td>37.16</td>
</tr>
<tr>
<td>15.00</td>
<td>15.53</td>
<td>15.71</td>
<td>0.69</td>
<td>39.56</td>
<td>0.567</td>
<td>118.51</td>
<td>68.71</td>
</tr>
</tbody>
</table>

© M. Iskander at NYU
Tension Crack

With depth of crack

Without depth of crack
Tension Crack

a) With depth of crack

\[ H_1 = 15.96 \, \text{m} \]
\[ 4.61 \, \text{m} \]

Actual Stress Distribution

Assumed Stress Distribution (2)

\[ P_{AEH} = 1496 \, \text{kN} \, \text{(Eq. 22)} \]

b) Without depth of crack

\[ H_1 = 15.96 \, \text{m} \]
\[ 5.32 \, \text{m} \]

Actual Stress Distribution

Assumed Stress Distribution (1)

\[ P_{AEH} = 1726 \, \text{kN} \]
• Depth of tension crack:

\[ z_c = H' \left( 1 - \frac{0.9\sigma_{AEH(z = H')}'}{\sigma_{AEH(z = H')} - \sigma_{AEH(z = 0.1H')}'} \right) \]

\[ H' = H \frac{\cos(\beta - \omega)}{\cos \beta \cos \omega} \]

• Horizontal thrust on wall:

\[ P_{AEH} = \frac{1}{2} \sigma_{AEH(z = H')} ' \left( H_l - z_c \frac{\cos \beta}{\cos (\beta - \omega)} \right) \]

\[ H_l = \frac{H}{\cos (\omega)} \]

More conservative

\[ P_{AEH} = \frac{1}{2} \sigma_{AEH(z = H')} ' H_l \]
Example

Horizontal Pseudo-Static Lateral Earth Pressure

\[ \gamma = 23 \text{ kN/m}^3 \]
\[ \varphi = 30^\circ \]
\[ c = 20 \text{ kPa} \]

\[ z_i = \frac{z_w}{\cos(\omega)} \]
\[ z = z_w \frac{\cos(\beta - \omega)}{\cos(\beta)\cos(\omega)} \]

\[ \alpha_h = k_g = 0.2 \text{g} \]
\[ \alpha_v = k_g = 0.1 \text{g} \]
\[ \omega = 20^\circ \]
\[ \theta = 15^\circ \]

\[ H = 15 \text{ m} \]

<table>
<thead>
<tr>
<th>( Z_w (m) )</th>
<th>Depth Along Length of Wall, ( Z_i (m) )</th>
<th>( Z (m) )</th>
<th>( J_0 ) (Eq. 13)</th>
<th>( \alpha_h ) (°) (Eq. 18)</th>
<th>( K_a ) (Eq. 16)</th>
<th>( \sigma^*_{ah} ) (kPa) (Eq. 15)</th>
<th>( \sigma^*_{AEH} ) (kPa) (Eq. 19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-11.55</td>
<td>-5.00</td>
<td>-9113</td>
<td>-23.01</td>
<td>-22.22</td>
</tr>
<tr>
<td>3.00</td>
<td>3.19</td>
<td>3.29</td>
<td>39.62</td>
<td>53.66</td>
<td>0.561</td>
<td>42.46</td>
<td>11.95</td>
</tr>
<tr>
<td>6.00</td>
<td>6.39</td>
<td>6.59</td>
<td>97.59</td>
<td>36.94</td>
<td>0.716</td>
<td>108.51</td>
<td>59.19</td>
</tr>
<tr>
<td>9.00</td>
<td>9.58</td>
<td>9.88</td>
<td>157.46</td>
<td>32.23</td>
<td>0.791</td>
<td>179.80</td>
<td>110.11</td>
</tr>
<tr>
<td>12.00</td>
<td>12.77</td>
<td>13.17</td>
<td>218.23</td>
<td>29.97</td>
<td>0.835</td>
<td>253.06</td>
<td>162.75</td>
</tr>
<tr>
<td>15.00</td>
<td>15.96</td>
<td>16.46</td>
<td>279.5</td>
<td>28.63</td>
<td>0.865</td>
<td>327.37</td>
<td>216.35</td>
</tr>
</tbody>
</table>
Example – Undrained Cohesive Backfill

\[ \gamma = 20 \text{ kN/m}^3 \]
\[ \varphi = 0^\circ \]
\[ C_a = 100 \text{ kPa} \]

\[ \sigma'_{aE} = 1.90 \pi z^2 + 10.44z - 188.58 \]
\[ R^2 = 0.998 \]

\[ H = 7.59 \text{ m} \]

\[ \alpha = k_0 \theta = 0.2g \]
\[ \alpha_0 = k_0 g = 0.1g \downarrow \]
\[ \alpha = 10^\circ \]
\[ \delta = 15^\circ \]

<table>
<thead>
<tr>
<th>( Z_m (m) )</th>
<th>Depth Along Length of Wall, ( Z_0 (m) )</th>
<th>( Z (m) )</th>
<th>( I_a ) (Eq. 13)</th>
<th>( \alpha_a (^\circ) ) (Eq. 18)</th>
<th>( K_a ) (Eq. 16)</th>
<th>( \sigma'_a ) (kPa) (Eq. 15)</th>
<th>( \sigma'_{aE} ) (kPa) (Eq. 19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-99.99</td>
<td>5.00</td>
<td>-95100</td>
<td>-199.24</td>
<td>192.45</td>
</tr>
<tr>
<td>2.00</td>
<td>2.03</td>
<td>2.09</td>
<td>-57.22</td>
<td>-0.73</td>
<td>-3.753</td>
<td>-157.21</td>
<td>155.16</td>
</tr>
<tr>
<td>4.00</td>
<td>4.06</td>
<td>4.19</td>
<td>-10.42</td>
<td>-11.55</td>
<td>-1.315</td>
<td>-110.09</td>
<td>110.15</td>
</tr>
<tr>
<td>6.00</td>
<td>6.09</td>
<td>6.28</td>
<td>41.24</td>
<td>-41.23</td>
<td>-0.519</td>
<td>-65.22</td>
<td>55.77</td>
</tr>
<tr>
<td>8.00</td>
<td>8.12</td>
<td>8.38</td>
<td>100.21</td>
<td>69.51</td>
<td>0.415</td>
<td>69.57</td>
<td>12.67</td>
</tr>
<tr>
<td>10.00</td>
<td>10.15</td>
<td>10.47</td>
<td>178.94</td>
<td>33.53</td>
<td>0.785</td>
<td>164.34</td>
<td>119.15</td>
</tr>
</tbody>
</table>
Comparison with Other Methods

Without wall and backfill inclination

With wall and backfill inclination

8/10/2016
• Normalization to facilitate comparison

\[ c^* = \frac{c}{\gamma H} \]

\[ P_{AE}^* = \frac{P_{AEH}}{\gamma H^2} \]

• Frictional-cohesive backfill
Direction of Vertical Acceleration

$P_{AE}^{*}/H^2$ vs. $K_h$

1. **Upward vertical acceleration**
   - $c^* = c/\gamma H = 0.05$
   - $\phi = 30^\circ$
   - $K_v = 0.1$ up

2. **Downward vertical acceleration**
   - $c^* = c/\gamma H = 0.05$
   - $\phi = 30^\circ$
   - $K_v = 0.1$ down

© M. Iskander at NYU
Natural soils often have some cohesion
Cohesion reduces lateral earth pressure
Conservative or unconservative?

Huntington (1957) noted: “For many years it was almost universal practice to compute the earth pressure against a retaining wall on the assumption that the soil was cohesionless and that the value of φ could be considered equivalent to the angle of repose…. The most common assumption according to this practice was that the slope of the angle of repose was 1.5 horizontal: 1 Vertical” (i.e., 33.41).
Design Charts
Vertical Wall and Horizontal Backfill

© M. Iskander at NYU
Design Charts
Inclined Wall and Backfill

© M. Iskander at NYU
Conjugate Stress Approach for Rankine Seismic Active Earth Pressure in Cohesionless Soils

Margad Iskander, Ph.D., P.E., F.ASCE, Mehdi Omidiar, S.M.ASCE, and Omar Elsherif, Ph.D., P.E.

Abstract: The Rankine tensionless earth pressure solution has been expanded for the calculation of seismic active earth pressure on rigid retaining walls in cohesionless soils. The expanded solution is based on the equilibrium concept, utilizing a conjugate stress analysis. A single integration uses the static and pseudo dynamic seismic analysis of active condition. The equation accounts for the effects of both static and dynamic, and can readily accommodate layered soil profiles and the presence of grouting, which requires stress analysis in a combined way. A simple stress expression for the constant value currently useful in practice, cannot be derived directly from the stress analysis in the form of a pseudo static analysis. The proposed model can be used in the design of rigid retaining walls based on a pseudo static analysis. Comparative studies with the McKinney-Mohr solution showed a reasonable agreement with which the solution presented in this paper can be used for the analysis of rigid retaining walls in cohesionless soils.

Keywords: Seismic pressure, Rankine method, Conjugate stress, Soil stiffness, Active earth pressure, Soil failure.

Introduction

The problem of seismic earth pressure behind retaining walls is typically solved by the equilibrium explained by Mohr and Muiravsky (1956) [held in ASCE JGGE (2006) 1206]. For the Rankine method, the equilibrium of the static stress can be used to eliminate the pseudo static analysis. The Modified Mohr solution has been extended by several authors to accommodate for the seismic stresses and the pseudo static pressure effect (e.g., Richarda et al. 1999; Kwan R.C. 1996; Stuia et al. 2000).

Finite element studies as Beant et al. (1987) state that the presence of the seismic load has been defined as a significant feature of the problem. In this study, the seismic earth pressure solution is extended to the Rankine method to account for the pseudo static analysis. The Rankine formulation considers the influence of the seismic load on the wall and the wall foundation. The solution is also applicable to the case of a semi-rigid wall, for which the wall is considered rigid, and the wall and the wall foundation is considered rigid in the case of the seismic load.
Developed an expressions for the static and pseudo-static seismic analyses of $c-\phi$ backfill.

Results are identical to those computed with the Mononobe–Okabe method for cohesionless soils, provided the same wall friction angle is employed.

For $c-\phi$ soils, the formulation yields comparable results to available solutions for cases where a comparison is feasible.
Thank You

Acknowledgments

• Mehdi Omidvar
• Chris Chen
• Omar Elsherif