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Taking an inner product with ψ_ℓ^0:

$$\sum_{m\neq n} (E_m^0 - E_n^0)\, C_m^{(n)} \langle \psi_\ell^0 | \psi_m^0 \rangle = -\langle \psi_\ell^0 | H' | \psi_n^0 \rangle + E_n' \overbrace{\langle \psi_\ell^0 | \psi_n^0 \rangle}^{\delta_{\ell n}}$$

If $\langle \psi_\ell^0 | \psi_m^0 \rangle = \delta_{\ell m}$ so nonzero when $\ell = m$ but left hand side will be zero when $\ell = n = m$. for $n \neq \ell$:

$$(E_\ell^0 - E_n^0)\, C_\ell^{(n)} = -\langle \psi_\ell^0 | H' | \psi_n^0 \rangle + 0 \Rightarrow C_m^{(n)} = \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}$$

so

$$\boxed{\psi_n' = \sum_{m\neq n} \frac{\langle \psi_m^0 | H | \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \psi_m^0}$$

* $E_n^0 \neq E_m^0$ as long as the unperturbed energy spectrum is nondegenerate

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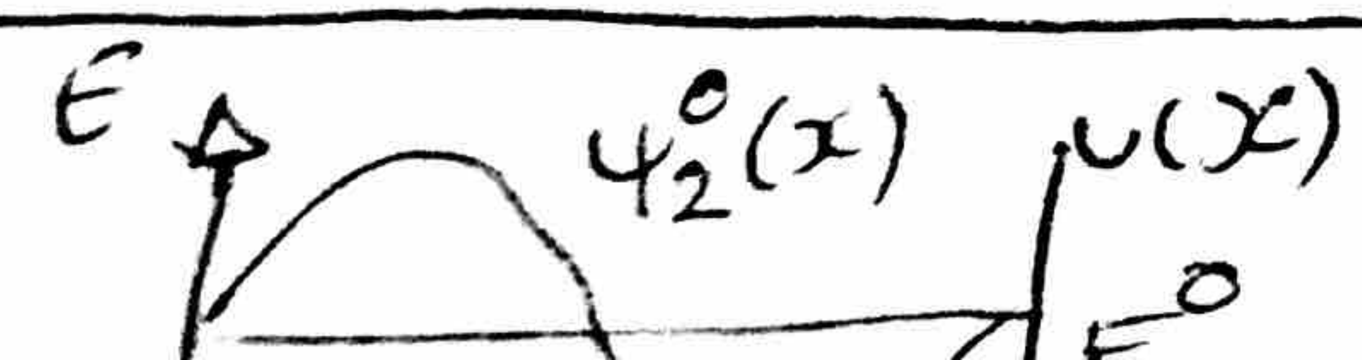
HW (Bonus)

$$|\psi_n'\rangle = \sum_{m\neq n} C_m^{(n)} |\psi_m^0\rangle = C |\psi_n^0\rangle$$

$\langle \psi_m^0 | \psi_n' \rangle = \langle \psi_m^0 | C | \psi_n^0 \rangle \Rightarrow$ what property operator C has? (Hemitian? unitary?
(or matrix) ...

*

Ex/



$$H = H_0 + H' = 0 + V_0\, \Theta\left(x - \frac{a}{2}\right)$$