HW2 Quantum Mechanics 2

1/ G. 7.3 : Two identical spin-zero bosons are placed in an infinite square well. They interact weakly with one another, via the potential

$$V(x_1, x_2) = -aV_0\,\delta(x_1 - x_2)$$

(where V_0 is a constant with the dimensions of energy, and a is the width of the well.

a/ First, ignoring the interaction between the particles, find the ground state and the first excited state – both the wavefunctions and the associated energies.

b/ Use first-order perturbation theory to estimate the effect of the particle – particle interaction on the energies of the ground state and the first excited state.

2/ Consider a particle of mass m in an infinite square well $V(x) = \begin{cases} 0 & -a<x<a \\ \infty & \text{otherwise} \end{cases}$

Now add a small perturbation to the potential $V'(x) = \begin{cases} -\lambda v & -a<x<0 \\ \lambda v & 0<x<a \end{cases}$

where λ is a small number. Treating $V'(x)$ as a small perturbation, use perturbation theory to find: (a) Eigenvalues to the second order of λ

(b) Eigenstates to the first order of λ.

3/ Consider a simple harmonic oscillator potential: $H_0 = \dfrac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$.

We add a small perturbation in the system $H' = \lambda x^3$

Use perturbation theory to find (a) eigenvalues to the second order of λ

(b) Eigenstates to the first order of λ.

4/ Consider a system with the following Hamiltonian

$$H = \begin{pmatrix} a_1 & 0 & b \\ 0 & a_2 & c \\ b & c & a_3 \end{pmatrix}$$

a) Use perturbation theory to find eigenvalues to the second order and eigenstates to the first order.

b) Consider the special case of $c = 0$. Find the exact eigenvalues and eigenstates. Compare these with the results from using perturbation theory.

c) Repeat parts (a) and (b) when $a_1 = a_2$.

5/ Suppose we perturb the infinite cubical well by putting a delta function
(7.11) "bump" at the point $(a/4, a/2, 3a/4)$: $H' = a^3 V_0 \delta(x - a/4)\delta(y - a/2)\delta(z - 3a/4)$

Find the first-order corrections to the energy of the ground state and the (triply degenerate) first excited states.