HW3

1/ G 8.6 (a): Use the variational principle to prove that the first-order non-degenerate perturbation theory always overestimates the ground state energy.

(b) Based on part (a), you would expect that the second-order correction to the ground state is always negative. Confirm that this is indeed the case by examining Equation: $E_n^2 = \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}$

2/ G 8.5 Using a trial function of your own devising, obtain an upper bound on the ground state energy for $V(x) = \begin{cases} mgx & x > 0 \\ \infty & x \leq 0 \end{cases}$

and compare it with the exact answer $E_{gs} = 2.338 \left(\frac{mg^2 \hbar^2}{2} \right)^{1/3}$

3. Calculate the ground state of a hydrogen atom using the variational principle. Assume that the variational wavefunction is a Gaussian of the form $Ae^{-(r/\alpha)^2}$ where A is for normalization and α is a variational Parameter. How does this variational energy compare with the exact ground state energy?

4. G 8.7: Using $E_{gs} = -79\,eV$ for the ground state energy of helium, Calculate the ionization energy (the energy required to remove just one electron). Hint: First Calculate the ground state energy of the helium ion, He^+, with a single electron orbiting the nucleus; then subtract the two energies.

HW3Bonus : why could we use a non-degenerate perturbation theory for hydrogen atom?