HW5, Quantum Mechanics 2

Problem: In Yukawa's original theory (1934) which remains a useful approximation in nuclear physics, the "strong" force between protons and neutrons is mediated by the exchange of π-mesons. The potential energy is $V(r) = -r_0 V_0 \frac{e^{-r/r_0}}{r}$ where r is the distance between the nucleons, and the range r_0 is related to the mass of the meson: $r_0 = \frac{\hbar}{m_\pi c}$. The Schrodinger equation for the proton /neutron system is $-\frac{\hbar^2}{2\mu}\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r)$ where μ is the reduced mass (proton and neutron have almost identical masses, m) and r is the position of the neutron (say) relative to the proton: $r = r_n - r_p$. Show that there exists a solution with negative energy (a bound state), using a variational trial wave function of the form: $\psi_B(r) = Ae^{-\beta r/r_0}$.

a) Determine A by normalizing $\psi_B(r)$. Note this is a 3D problem.

b) Find expectation value of the Hamiltonian in the state ψ_B. Express your answer in terms of $\gamma = \frac{2\mu r_0^2}{\hbar^2} V_0$

c) optimize your wave function by $\frac{dE(\beta)}{d(\beta)} = 0$. Find E_{min} by eliminating γ in favor of β and express E_{min} a function of β.