So $\langle H \rangle = E_1 - 2|A|^2 \left(\frac{e^2}{4\pi\epsilon_0}\right)\left[\langle \psi_0(r_1)\left|\frac{1}{r'}\right|\psi_0(r_1)\rangle + \langle \psi_0(r)\left|\frac{1}{r}\right|\psi_0(r')\rangle\right]$

$D = a\langle \psi_0(r_1)\left|\frac{1}{r'}\right|\psi_0(r_1)\rangle$ Direct integral

$X = a\langle \psi_0(r_1)\left|\frac{1}{r}\right|\psi_0(r')\rangle$ Exchange integral

} Bonus HW

$$D = \frac{a}{R} - \left(1 + \frac{a}{R}\right)e^{-2R/a}$$

$$X = \left(1 + R/a\right)e^{-R/a}$$

$$\Rightarrow \langle H \rangle = \left[1 + 2\frac{D+X}{1+I}\right]E_1$$

↑ for electrons

$$V_{PP} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{R} = +\frac{-2a}{R}E_1$$

↑ for protons