Quantum Mechanics 2
Spring 2019

Homework 4 Problems

1. (a) Suppose you put both electrons in a Helium atom with $n = 2$. What would the energy of the emitted electron be?

   Solution: In this situation, one electron will drop down to the ground state and the other electron will gain the same amount of energy that the other lost. The electron that goes from the $n = 2$ to the $n = 1$ state loses energy $\Delta E = E_2 - E_1$ where the energy levels are those of the helium ion (we are ignoring electron-electron interactions). For the helium ion the energy levels are:

$$E_n = -\frac{4}{n^2} E_{\text{Ryd}} \tag{1}$$

   where $E_{\text{Ryd}} = 13.6eV$ is the magnitude of the energy of the hydrogen ground state. In general, the energy eigenvalues of a one electron ion with a nucleus with Z protons is Z^2 times the hydrogen atom energy levels. For the helium ion, $Z = 2$. Therefore $\Delta E = 40.8eV$. The other electron gains that amount of energy. It's final energy is therefore, $E_f = E_2 + \Delta E = -13.6eV + 40.8eV = 27.2eV$.

   (b) Describe (quantitatively) the spectrum of the helium ion.

   Solution: The energy spectrum of He^+ is discussed in the previous problem and given explicitly in (1).

2. Discuss (qualitatively) the energy spectrum of helium if the electrons (a) are identical bosons or (b) distinguishable particles. Assume the electrons still are spin-$\frac{1}{2}$ (i.e. 2 spin states) and pretend there is no relationship between spin and statistics (i.e. for part a, we can have electrons with 2 spin states obeying Bose-Einstein statistics).

   (a) Identical Bosons

Solution: The idea here is that the total wavefunction must be symmetric (spatial part plus spin part). In this case, whenever we are in a state where both electrons have energy and spatial quantum numbers (i.e. same n, l, and m), then the spatial wave-function necessarily symmetric which means for bosons, the spin part must also be symmetric. There are 3 symmetric spin states and 1 anti-symmetric for spin-$\frac{1}{2}$. Thus, the ground state and all states where both bosons have the same spatial quantum numbers have a 3-fold degeneracy. Whenever the state is a superposition of two different spatial states, we have a 4-fold degeneracy (3 states with symmetric spatial and symmetric spin plus one state with anti-symmetric spatial and anti-symmetric spin).

(b) Distinguishable Particles

Solution: In this case, there is no restriction on the parity of the wavefunction under an exchange of the particles so states with identical spatial quantum numbers have a 4-fold degeneracy and states with different spatial quantum numbers have an 8-fold degeneracy.