Noisy Memory and Asset Price Fluctuations

Yeji Sung

Columbia University

2019 Sloan-Nomis Workshop
Introduction

- Why are asset prices so volatile and sensitive to news?
  - Shiller (1981), De Bondet and Thaler (1987), Cutler, Poterba, and Summers (1990), and more

- How to model belief formation of asset prices?
  - One extreme: Rational Expectations + Full Info
  - The other extreme: Expectations separate from the model
    - e.g. Extrapolative belief based on statistical projection

- A tractable model that lies in between?
What I do

▶ de Silveira and Woodford (2019)

▶ With a limit on the complexity of memory, more recent news will be given disproportionate weight

▶ A simple explanation for why consumption over-reacts to news

▶ Research question:

Can the theory of noisy memory predict a realistic asset prices dynamics?
I show it can

- Compared to perfect memory rational expectations,
  - asset prices are more sensitive to news
  - asset prices are more volatile and serially correlated
  - trade volume is larger

despite assuming a very low dimensional underlying shock
A model of noisy memory

- DM observes a realization of
  \[ d_t \sim \mathcal{N}(\mu, \sigma_d^2) \quad i.i.d \]
  \( \sigma_d^2 \) is known, but \( \mu \) is not known

- DM has a prior distribution for \( \mu \)
  \[ \mu \sim \mathcal{N}(0, \omega) \]

- There's a limit on the precision of memory
  - DM has imperfect memory of the past \( d_{t-j} \)
    - Past observations are accessible only through memory
  - DM optimally chooses a memory state \( \{m_t\} \)
Storing memory is costly

- Cost of storing a memory
  - \( l_t = \theta \times \text{mutual info between } (m_t, d_t) \text{ and } m_{t+1} \)

- \( m_{t+1} \) is a noisy function of \( (m_t, d_t) \)
  - If \( m_{t+1} \) is not informative about \( (m_t, d_t) \), \( l_t = 0 \)
  - If \( m_{t+1} \) is completely informative about \( (m_t, d_t) \), \( l_t = \infty \)
Consumption and portfolio choice

DM maximizes

\[ \mathbb{E} \sum_{j=1}^{\infty} \beta^j \left[ - \exp(-\alpha c_t) - \theta l_t \right] \]

subject to

\[ c_t + p_t \theta_{t+1} + \theta_{rf}^t = p_t \theta_t + R \theta_{rf}^t, \quad \forall t \]

- \( p_t \): price of risky assets
- \( \theta_{t+1} \): holdings of risky assets
- \( \theta_{rf}^t \): holdings of risk-free assets
Optimal portfolio choice

- More risky assets if high expected returns and low variance
  \[
  \theta_{t+1}^i = \frac{E_t^i \rho_{t+1} + d_t - R\rho_t}{\kappa V_t^i \rho_{t+1}}
  \]
  where \( \kappa \equiv \alpha \frac{R-1}{R} \)

- DM forms subjective beliefs based on cognitive states \((m_t^i, d_t)\)

- Memory is used to improve an estimate of \(\mu\)
  \[
  \mu|m_t^i \sim \mathcal{N}(m_t^i, \sigma_\pi^2(t))
  \]
Optimal memory choice

- Optimal estimate of $\mu$

\[
\mathbb{E}[\mu|m_t^i, d_t] = (1 - \gamma(t)) m_t^i + \gamma(t) d_t
\]

with gain coefficient $\gamma(t) \equiv \frac{\sigma_{\gamma}^2(t)}{\sigma_{\gamma}^2(t) + \sigma_d^2}$

- Optimal memory stores a noisy record of $\mathbb{E}[\mu|m_t^i, d_t]$

\[
m_{t+1}^i = \lambda(t) \mathbb{E}[\mu|m_t^i, d_t] + \omega_{t+1}
\]

where $\omega_{t+1} \sim \mathcal{N}(0, \sigma_{\omega}^2(t + 1))$

- We numerically solve for $\{\gamma(t)\}$, which completely determines $\{\lambda(t)\}$ and $\{\sigma_{\omega}^2(t + 1)\}$
Gain coefficient in the long run

- When $\theta = 0$, $\gamma(t) \to 0$
  - Precision $\left(\frac{1}{\sigma^2 \pi}\right)$ linearly increases over time
- When $\theta > 0$, $\gamma(t) \to \gamma > 0$
Consequences of $\gamma > 0$ on asset prices

- Price of risky assets in equilibrium (assuming fixed supply)

$$p_t = p_0(\theta) + p_m(\theta)\bar{m}_t + p_d(\theta)d_t$$

- $\bar{m}_t \equiv \int m_t^i di$
- $p_d(\theta)$ increases in $\theta$
- By backward substitution of $\bar{m}_{t+1} = \lambda[(1 - \gamma)\bar{m}_t + \gamma d_t]$, 

$$p_t = p_0(\theta) + p_m(\theta)\lambda\gamma \sum_{j=0}^{t} \rho^j d_{t-1-j} + p_d(\theta)d_t$$

where $\rho \equiv \lambda(1 - \gamma)$

- When $\theta = 0$, small fluctuations + i.i.d process
- When $\theta > 0$, big fluctuations + long lag!
Asset prices over-react to news of dividend
Dividend forecasts extrapolate news of dividend

Change in average dividend forecasts

- $\hat{\theta} = 0.00$
- $\hat{\theta} = 0.05$
- $\hat{\theta} = 0.10$
- $\hat{\theta} = 0.15$
- $\hat{\theta} = 0.20$
Asset prices are more volatile and serially correlated

Price of risky assets

- Standard deviation
- Auto-correlation
Excess returns are higher and more volatile
Asset holdings distribution is non-degenerate

Risky Assets Holdings

Trade volume
Trade volume comparison
Conclusion

- Theory of noisy memory can predict a realistic feature of asset price fluctuations

- Optimal Bayesian Inference is yet to be rejected as a useful modeling tool

- Future work includes
  - Empirically more relevant dividend process
  - Incorporation into a quantitative model
Asset prices are more volatile
Asset prices are more serially correlated
Excess returns are higher
Excess returns are more volatile
Trade volume is larger

![Graph showing trade volume as a function of \( \hat{\theta} \). The graph indicates an increase in trade volume with an increase in \( \hat{\theta} \).]
Asset holdings distribution is more centered to zero.