Excess Volatility from Increasing Overreaction

Daniele d’Arienzo

Bocconi University - Harvard University

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Motivations

Excess volatility and Beliefs

Conclusions
**Motivations**

- Shiller (1981): excessively volatile prices. Due to beliefs?
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- a constant discount rate is assumed: discount rates movements or excess beliefs movements?
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  • beyond discount rates
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- Rational Expectations impose constraints across maturities

\[ P_{t,m} = \mathbb{E}_t^Q [P_{t+1,m-1}] = \mathbb{E}_t^Q \left[ \mathbb{E}_{t+1}^Q [P_{t+2,m-2}] \right] = \ldots \]
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- Long maturity yields $\approx$ Expected future short maturity yields

Factors = 3, $R^2 = 100.0\%$

Price Volatility (daily)

Maturity (years)
Research Question

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3. Explain increasing over-reaction and excess volatility with a model of diagnostic expectations
   - key ingredient: agents over-react more in more volatile environments
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Testing for Rational Expectation

- \( P_{t,m} \) := price of a ZCB with maturity \( m \) at time \( t \)
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- Forecast error $FE_{t+1}^m := y_{t+1,m} - \hat{y}_{t+1|t,m}$, where $\hat{y}_{t+1|t,m}$:

\[\hat{y}_{t+1|t,m}:\]
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  - directly observed for survey data; or

\( \hat{y}_{t+1|t,m} \) inferred from prices (Ross Recovery Theorem)
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- Forecast revision $FR_t^m := \hat{y}_{t+1|t,m} - \hat{y}_{t+1|t-1,m}$
Testing Under/Over-reaction to Information

Coibion and Gorodnichenko (2015) regression:
\[ FE^m_{t+1} = a_m + b_m FR^m_t + \epsilon^m_{t+1} \]

- \( b_m > 0 \): under-reaction to information
- \( b_m < 0 \): over-reaction to information

• both survey and recovered forecast exhibit increasing over-reaction
• recovered beliefs display under-reaction at short end, over-reaction at long end
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- both survey and recovered forecast exhibit increasing over-reaction
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survey data and recovered beliefs are strongly correlated
departures at short maturities: heterogeneity?
Diagnostic Expectations

Empirical facts

- excess volatility
- increasing over-reaction
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Empirical facts

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Modeling Asset Prices and Beliefs

- One factor drives the yield curve dynamics
  \[ X_{t+1} | X_t \sim \mathcal{N}(\rho X_t, \sigma^Q) \]
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\[
\underbrace{\mathbb{Q}^\theta(X_{t+1}|X_t)}_{\text{Diagnostic Probability}} \propto \underbrace{\mathbb{Q}(X_{t+1}|X_t)}_{\text{Objective Probability}} \left( \frac{\mathbb{Q}(X_{t+1}|X_t)}{\mathbb{Q}(X_{t+1})} \right)^\theta
\]

Representativeness
Affine models under Rational and Diagnostic Expectations

- Under **Rational Expectations**:

\[ y_{t,m} = a_m + \left(1 + \rho + \ldots + \rho^{m-1}\right) X_t \]
Affine models under Rational and Diagnostic Expectations

• Under Rational Expectations:

\[ y_{t,m} = a_m + (1 + \rho + \cdots + \rho^{m-1}) X_t \]

\[ \frac{V^Q[y_{t,m}]}{V^Q[y_{t,1}]} = \left( \frac{1 - \rho^m}{1 - \rho} \right)^2 \]
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- **Under Diagnostic Expectations:**
  \[
  y_{t,m} = a^{\theta}_m + \frac{1 + \theta}{1 + \theta - \theta \left(\frac{\sigma_m}{\sigma_{Q}}\right)^2} \left(1 + \rho + \cdots + \rho^{m-1}\right) X_t
  \]
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- **Under Rational Expectations:**

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- **Under Diagnostic Expectations:**

  \[ y_{t,m} = a^\theta_m + \frac{1 + \theta}{1 + \theta - \theta \left(\frac{\sigma_m}{\sigma_\infty}\right)^2} \left(1 + \rho + \cdots + \rho^{m-1}\right) X_t \]

  \[
  \frac{\mathbb{V}_Q^\theta[y_{t,m}]}{\mathbb{V}_Q^\theta[y_{t,1}]} = \left(\frac{1 + \theta}{1 + \theta - \theta \left(\frac{\sigma_m}{\sigma_\infty}\right)^2}\right)^2 \left(\frac{1 - \rho^m}{1 - \rho}\right)^2
  \]
Excess Volatility: Affine Three Factor Model

- **Unconstrained**
- **Diagnostic Expectations**
- **Rational Expectations**

Diagram showing excess volatility over maturity with three different expectations. The x-axis represents maturity, and the y-axis represents excess volatility. The graph compares unconstrained, diagnostic expectations, and rational expectations over varying maturity periods.
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- Both survey data and beliefs retrieved from Ross recovery theorem exhibit increasing overreaction

- Rationalize both increasing over-reaction and excess volatility within diagnostic expectations. Key ingredient: agents over-react more in more volatile environments