Multidimensional and Selective Learning
Sloan-Nomis Workshop

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Motivation

- DM needs to choose multiple attributes/inputs
- Any arbitrary correlation allowed between attributes
- Optimal learning strategy: what and how much to learn?
- Examples: agricultural input choices, job assignment etc.
Why Multidimensional Decision Problem?

- **Selective Learning**: Productivity of only a subset of attribute learned separately (Hanna et al 2014, Bloom et al 2014 etc)
- Belief about correlation affects learning choice
Research Agenda

- Solve for the optimal learning strategy in the multidimensional choice setting
- Find conditions under which selective learning is optimal
- Policy Implication: optimal information provision
  - extension services
  - management practices training
Choice Problem: \( n = 2 \) inputs

- \( A = X \times Y \); action/decision space
- \( X = \{x_1, x_2, \ldots x_n\}, \ Y = \{y_1, y_2, \ldots y_n\} \): two inputs
- \( Y = \{0, 1\} \): output
- \( \pi : A \rightarrow Y \): payoff function
- \( \Omega \): state space (set of all possible payoff function),
- \( \omega \): one realization of \( \pi \), typical state
- \( \mu_0 \in \Delta(\Omega) \): prior
- \( \mu^* \): true state; \( supp(\mu^*) \subseteq supp(\mu_0) \)
Learning Technology

Table 1: Example of payoff matrix/state

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>( \frac{1}{3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
</tbody>
</table>

- DM can uncover any cell for a fixed cost of \( c_l \): observe 0 or 1
- DM can uncover any average for a fixed cost of \( c_a \): observe the true row or column average
- DM can open any number of cells or average in any order sequentially
- Bayesian updating
Properties of belief

- Expected payoff given belief $\mu_t : \pi_t$
- Uncertainty given belief $\mu_t$: $H(\mu_t)$
  i. $H(\mu_t)$: Shannon entropy of belief at round $t$

\[ H(\mu_t) = - \sum_{\omega \in \Omega} \mu_t(\omega) \ln \mu_t(\omega) \]

ii. $H(\mu_t) \in \mathbb{R}_+$
Decision Problem

Learning strategy

Learning strategy specifies a conditional sequence of observations \( \gamma(\mathcal{P}) \) such that observation of \( t^{th} \) round depends on the belief after of the \((t - 1)\) observations.

Decision Problem

DM chooses a learning strategy to maximize his expected payoff subject to the cost of learning,

\[
W(\mu_0) = \max_{\gamma(\mathcal{P})} E \left[ \pi(a_{ij}) - c(\mathcal{P}) \middle| \gamma, \mu_0 \right]
\]  
(DP)
Recursive Problem: Main Results

1. When is learning optimal?
   - Uncertainty is in an interval, i.e., not too low or too high

2. Whether to observe a cell of an average?
   - Higher uncertainty $\Rightarrow$ observe a average
   - Lower Uncertainty $\Rightarrow$ observe a cell

3. Which cell or average to observe?
   - Cell: highest one-round ahead expected payoff
   - Average: Reduces most uncertainty
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Stopping problem: Main results

- **Optimal learning strategy**: start with averages then switch to cell permanently
- **Selective Learning**: optimal if averages are sufficiently informative and learning is costly
- **Policy Implication**: reducing only one cost can decrease learning
Alternate Mechanism

- Sequential search: informationally inefficient
- Optimal Categorization: no bias-variance trade-off
Higher Dimensions: General Cost functions?

- For $n > 2$: scalability issues
- **Research Question**: does there exist a cost of learning function that is observationally equivalent to the prescribed learning mechanism?
- Symmetric Prior: Shannon entropy
Breadth vs Depth: Tree algorithms

- For $n > 2$ tree representation more tractable
- Tree structure assumes sequence of observation