Ideal Color Space for Reconstruction of Contaminant Transport in Transparent Soils

PhD Defense

By
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Objectives

Apply transparent soil as medium to study NAPL transportation mechanisms

Use ideal color space to correlate the image pixel information with the degree of concentration of NAPL inside a transparent porous medium

Estimate the volume of contamination plume only using a 2D projection of the model from its boundary

Reconstruct 3D map of NAPL volume distribution using three 2D orthogonal projections
Transparent Soil
Transparent soils are made by matching the refractive index of the soil particles and pore fluid to prevent refraction of light.

**Soil Particles**
- Guzman & Iskander, 2012
- Ezzein & Bathurst, 2011

**Pore Fluid**
Two Immiscible fluids can be used:

1. **Sucrose water:**
   Low Color Sucrose™ (LCS)

2. **Mineral oil:**
   Mixture of Krystol 40™ and Puretol 7™

⚠️ **Refractive index at 25°C:**
1.458
Methodology

Data Acquisition
2D projection of 3D model at its boundaries

Data Preparation
Find and informative color space and remove noises
using a proper filter

Correlation function
Estimate plume width using color space component values

NAPL Volume Calculation
Calculate volume of injection using 2D projection values

3D Reconstruction
Applying discrete tomography to reconstruct 3D model from its 2D projections
Research Phases

Phase I
Calibration Model

Phase II
Validation Model

Phase III
Final Model
Image Acquisition
Color Space

- A three dimensional space that can be used to geometrically represent any specific color

Color Space Families

1. Primary spaces, based on trichromatic theory which assumes that each color can be represented by a specific amount of three primary colors
2. Luminance–chrominance spaces which consist of one luminance and two chrominance components
3. Perceptual spaces, based on human color perception
4. Independent axis spaces, based on statistical independency of components
Luminance-Chrominace Space

Luminance:
Luminance is the intensity of light emitted from a surface per unit area in a given direction.

Chrominance:
Chrominance is an objective specification of a color regardless of its luminance.
Common Color Spaces

**Primary spaces**
- Real Primary space (Images Acquisition) $(R,G,B)$
- Imaginary Primary space (Intermediate Transformation) $(X,Y,Z)$

**Chromaticity spaces**
- $(r,g,b)$
- $(x,y,z)$

**Luminance-Chrominance spaces**
- Digital Coding space $(Y,Cb,Cr)$
- Television spaces $(Y,I,Q)$, $(Y,U,V)$
- Perceptually Uniform Spaces $(L,a,b)$, $(L,u,v)$

**Perceptual spaces**
- Spherical Coordinate space $(L,M,S)$
- Polar Coordinate space $(L,C,H)$
- Cylindrical Coordinate spaces $(H,S,I)$, $(H,S,L)$, $(H,S,V)$
Concentration vs. Color Space Component
Concentration vs. Color Space Component
Beer-Lambert law used to obtain correlation between dye concentration, plume length, and intensity of color in any color space component

Beer-Lambert Law:
There is a relation among the transmittance, \( \tau \), which is the ratio of the intensity of the transmitted beam, \( I_t \), to the incident beam, \( I_0 \), as it travels through a medium, the concentration of solute, \( C \), the distance that the light travels through the solution, \( l \), and the extinction coefficient, \( \varepsilon \)

\[
\tau = \frac{I_t}{I_0} = 10^{-\varepsilon LC}
\]

4 compartments full with dye concentration of \( C \) is taken as 1 compartment full with 4\( C \) concentration

\[
\begin{align*}
\{ & l = 4 \text{ filled} \quad \{ & l = 1 \text{ filled} \\
C = 200 \text{ ppm} & \} \quad C = 800 \text{ ppm}
\end{align*}
\]
Calibration Model 1

- **Color dyes**
  1. Red
  2. Blue
  3. Yellow
  4. Green
  5. Neon Purple
  6. Neon Pink
  7. Neon Green
  8. Neon Blue

- **Dyes Concentrations**
  1. 2000 ppm

- **Color Spaces**
  1. RGB
  2. XYZ
  3. Lab
  4. YC\textsubscript{B}C\textsubscript{R}
  5. YIQ
  6. HSI
Calibration Model 1

- RAW data acquisition from camera
- Conversion to TIFF using dcraw
- 100 x 100 pixel interrogation window selection
- Color space transformation
- Background correction
- Conversion to mean and standard deviation for data visualization
- Filter application, evaluation, and selection
- Normalization of data
- Concentration range selection
- Regression analysis for color component value vs. dye concentration
- Evaluation of components based on PSNR values
- Best color dye selection
- Best color space component selection

Initial Model

Step I

Step II

Step III

Step IV

Step V

Step VI

Step VII

Step VIII

Original images

No filter

Average Filter Size: 15 Pixel
Data Presentation
Effect of Filtering

- Convolution Average Filter
- Gaussian Filter

Red

- Standard Deviation
- Filter Block Size

Blue

- Standard Deviation
- Filter Block Size

Yellow

- Standard Deviation
- Filter Block Size

Images show the effect of different filters on color channels for various filter block sizes.
## Peak of Signal to Noise Ratio (PSNR)

**Mathematical Formulation:**

\[
PSNR = 10 \log \frac{\psi^2_{max}}{\sigma_e^2}
\]

\[
\sigma_e^2 = \frac{1}{N \times M} \sum_{i=1}^{N} \sum_{j=1}^{M} (y_{i,j} - f_i)^2
\]
Effect of noise on PSNR value
PSNR Results

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</table>

0 16 20 24 28 40
**Key Points - Calibration Model 1**

- **Best components** for color dyes belonged to **chromatic** families of color space including HSI, YCBCR, and Lab.

- **Luminance** components (grayscale) were generally **less useful** in rendering concentration, particularly **when averaging** was employed.

- Employing a **15x15 convolution** average markedly **improved** the efficacy of rendering concentration.

- **Green dye** and **CR** of YCBCR yielded the **maximum PSNR** value, 36.9 dB, among all evaluated color components and color dyes with the application of a **convolution average** filter.
Difficulties - Calibration Model 1

- Real Transparent soil was not used
- Porosity was not considered
- Effect of cell position in model was not considered
Calibration Model 2

- Color dyes
  1- Green
  2- Red
  3- Neon Purple
- Dyes Concentrations
  \(10^2 - 10^4\) ppm

- Color Spaces
  1- RGB
  2- XYZ
  3- Lab
  4- YC_B C_R
  5- YIQ
  6- HSI
  7- rgb
  8- xyz
Color space component value data (mean and standard deviation) vs. logarithm of concentration for green dye.
3D Regression Analysis

Relationship between color space component value and concentration of green dye a) 3D regression b) and c) 2D regression
PSNR Results - 3D Regression
PSNR Results - 2D Regression
**Key Points - Calibration Model 2**

- **Green dye** was best among the evaluated colors for relating pixel information to concentration.

- **Three dimensional** regression analysis produced higher PSNR values.

- Employing a **convolution average filter** improved the efficacy of rendering concentration for **red** and **neon purple** dyes, but not the green dye.

- The best efficacy of representing concentration was obtained using the **green dye** sucrose saturated fused quartz and color space component of **a (Lab)** and **X (XYZ)** in 2D regression analysis and **SI(HSI)** in 3D regression analysis.
3D Reconstruction

- Finding the volume of injection
- Spatial mapping of contamination using Binary Tomography
Binary Tomography
Binary Tomography
Binary Tomography
Reconstruction Algorithms

1. **Back Projection**
   - Simple Back Projection
   - Modified Back Projection

2. **Iterative Reconstruction**
   - Algebraic Iterative Reconstruction (AIR)
   - Stochastic Iterative Reconstruction (SIR)
Back projection is spreading the intensity of the projected data back through the slice equally, to achieve a rough approximation of the original slice.
Iterative Reconstruction

- An inverse problem
- Approximation of the solution
- The model initially assumed and after compare with the original projection.
- The cost function (error) would be minimized using an optimization model:
  \[ ||Ax-y||^2 \rightarrow \text{min} \]

Advantages:
- Improved insensitivity to noise
- Reconstructing an optimal image in the case of incomplete data
- Works when there is not have a large set of projections available
- Allows the use of improved regularization techniques
Problem Complexity

In General Problem:

Unknown Parameter:

\[ N_p = l^3 \]

Parametric Equations

\[ N_{eq} = 3l^2 \]

1×1×1 Model:

✓ 1×3 = 3 Equations for \( l^2 = 1 \) Parameter

2×2×2 Model:

✓ 2×3 = 6 Equations for \( 2^2 = 4 \) Parameters

3×3×3 Model:

✓ 3×3 = 9 Equations for \( 3^2 = 9 \) Parameters

4×4×4 Model:

✗ 4×3 = 12 Equations for \( 4^2 = 16 \) Parameters
Problem Simplification

hv-convex binary matrices:

- Concentration of NAPL is constant
- NAPL will continuously disperse around the point source

![h-convex](image), ![v-convex](image), ![hv-convex](image)

Integrated concentration

- Beer-Lambert law

\[
a_{i,j} = \sum_{k=1}^{p} x_{i,j,k}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n
\]

\[
b_{i,k} = \sum_{j=1}^{n} x_{i,j,k}, \quad i = 1, \ldots, m, \quad k = 1, \ldots, p
\]

\[
c_{j,k} = \sum_{i=1}^{m} x_{i,j,k}, \quad j = 1, \ldots, n, \quad k = 1, \ldots, p
\]
3D Carving Algorithm

1. Finding the exact position of the contaminant from projected images
2. Transferring this contaminant voxel information to an empty 3D reconstruction model
3. Removing the effect of the found contaminant concentration from all three projections
4. Defining position of the remaining (not reconstructed) NAPL of the 3D model using Back Projection Concept
3D Carving Algorithm

Read $A, B, C$ (orthogonal projection matrices);

$x = 0$; (zero 3D arrays)

find $a_{ij}$ such that $a_{ij}$ is maximum;

Max = $a_{ij}$

find $k_1, k_2, \ldots, k_F$ such that $b_{ik,j} \geq 1$

For $f=1$ to $F$, if $F \leq a_{ij}$ and $c_{ijk,j} \geq 1$

YES

NO

$a_{ij} - F$; $b_{ik,j} = b_{ik,j} - 1$;

$c_{ijk,j} - c_{ijk,j} - 1$; $x_{ijk,j} = 1$;

find $k_1, k_2, \ldots, k_G$ such that $c_{ijk,G} \geq 1$

For $g=1$ to $G$, if $a_{ij} = \text{Max}$ and $G \leq a_{ij}$ and $b_{ik,G} \geq 1$

YES

NO

$a_{ij} - G$; $b_{ik,G} = b_{ik,G} - 1$;

$c_{ijk,G} - c_{ijk,G} - 1$; $x_{ijk,G} = 1$;

if $a_{ij} = \text{Max}$

YES

NO

$a_{ij} - \text{Penalty}$

Redo for $B$ and $C$

if $a_{ij} \leq 0$

YES

Return $x$ (3D array)

Coordinate system
Defining Position of the Remaining NAPL

1. three projections of 3D reconstructed model with exact NAPL position is subtracted from the actual three projections. This subtraction results in three projections of the remaining NAPL.

2. In a new 3D model, these projections will be spread back equally to the potential position of the NAPL. The potential position of NAPL can be defined in two sub-steps.
   (i) Reconstructing a new 3D model via aforementioned algorithm utilizing stretched versions of the original projections that are magnified by factor of up to three
   (ii) subtracting the exact position model from this new 3D model.

3. The exact model and back projected potential model are combined into the final 3D reconstructed model and binarized by replacing all voxel values above a determined threshold with 1 and the rest with 0. This threshold is iteratively adjusted by comparing the original projection data and updating the 3D reconstructed model based on the difference between the estimated and the original projection.
Validation Model

Objective:
• Validating the developed tomographic routines by comparing the 3D estimated model and the original one
Noise Reduction using oil Submerging
Validation Model
Calibration Model

\[ N = 0.013493 \times 10^{8.55r} \]
Calibration Model

RGB Images  
r (rgb) images  
2D Projections  
3D Reconstruction

A  
B  
C  

0 1 2 3 4 5
3D Carving Reconstruction

Original 2D Reconstruction

Reconstructed 3D Model
Reconstruction Results

<table>
<thead>
<tr>
<th>Resolution</th>
<th>Original Projections</th>
<th>2D Projection of 3D Model</th>
<th>Error Image of projection</th>
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## Reconstruction Results

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<th>Estimated No. of Yellow Cubes</th>
<th>Error in estimated no. of Yellow Cubes (%)</th>
<th>Method</th>
<th>Penalty</th>
<th>Number of Iteration</th>
<th>Not Reconstructed Yellow Cubes (%)</th>
<th>Yellow Cube Reconstruction Error (%)</th>
<th>Total Reconstruction Error (%)</th>
<th>Best Yellow Cube Reconstruction Error (%)</th>
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<td>15.97%</td>
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<td>0.000047%</td>
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</tbody>
</table>
Transparent Soil Model

KEY Steps:

- 3 injection of 15mL=45mL @speed of 1mL/min
- 3 extraction of 150mL=450mL @speed of 10, 20, and 30 mL/min
NAPL Extraction Setup
Image Acquisition Setup

A

B

C
3D Reconstruction Flowchart

**DATA CONDITIONING**
- RAW data acquisition from camera
- Conversion to TIFF using dcraw
- Image Registration & Scaling
- Color space transformation
- Filter application of 15x15 pixel interrogation windows
- Image Cropping
- Background Correction
- Image Segmentation using $C_R$ of $Y_CbCr$
- Application of Binary Mask
- Obtain Concentration Profile using a of Lab

**2D VOLUME ESTIMATION**
- Determine integrated concentration using calibration curve
- Estimate NAPL depth based on number of NAPL voxels for each projection

**3D RECONSTRUCTION**
- Create a 3D model using all available data
- Carve out certain NAPL voxel positions
- Create a new 3D model using all available data, except certain NAPL voxel positions
- Carve out certain water voxel positions
- Create 2D projections of the two 3D potential NAPL voxel models
- Use Back Projection to estimate remaining NAPL voxel locations
Image Registration

<table>
<thead>
<tr>
<th>Side</th>
<th>Before injection</th>
<th>After 45 mL Injection</th>
<th>After 450 mL removal</th>
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<td>A</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>B</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>C</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Color Space Component Value vs. Pixel Location

<table>
<thead>
<tr>
<th>Comp.</th>
<th>RGB</th>
<th>YCbCr</th>
<th>Lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="RGB" /></td>
<td><img src="image2" alt="YCbCr" /></td>
<td><img src="image3" alt="Lab" /></td>
</tr>
</tbody>
</table>
# Color Space Analysis Steps

<table>
<thead>
<tr>
<th>Side</th>
<th>1. True RGB</th>
<th>2. Intensity of $C_a$ ($Y_aC_a$)</th>
<th>3. Binary Mask</th>
<th>4. Intensity of $a$(Lab)</th>
<th>5. Concentration (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
</tr>
<tr>
<td>B</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
</tr>
<tr>
<td>C</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Volume Estimation

\[ V_{\text{NAPL}} = \sum_{i=1}^{m} \sum_{j=1}^{n} N_{i,j} \times L^3 \]

\[ N_{i,j} = \frac{C_{i,j}}{c} \times \frac{n_{\text{Cal}}}{n_{\text{Model}}} \times \frac{D_{\text{Cal}}}{L} \]

\[ C_{i,j} = C_0 \times 10^{m \times x_{i,j}} \]
## Result - Volume Estimation

<table>
<thead>
<tr>
<th>Vol.</th>
<th>Side</th>
<th>Actual Volume Injected (mL)</th>
<th>Estimated Volume Based on Reconstruction (mL)</th>
<th>Model Error Based on NAPL Volume (%)</th>
<th>Model Error Based on Three Faces (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Y(YC&lt;sub&gt;B&lt;/sub&gt;C&lt;sub&gt;R&lt;/sub&gt;) a(Lab)</td>
<td>Y(YC&lt;sub&gt;B&lt;/sub&gt;C&lt;sub&gt;R&lt;/sub&gt;) a(Lab)</td>
<td>Y(YC&lt;sub&gt;B&lt;/sub&gt;C&lt;sub&gt;R&lt;/sub&gt;) a(Lab)</td>
</tr>
<tr>
<td>15</td>
<td>A</td>
<td>15</td>
<td>34 15.6</td>
<td>125.9% 3.9%</td>
<td>121.9% 3.4%</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>33 15.5</td>
<td>122.8% 3.3%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>33 15.5</td>
<td>116.9% 3.0%</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>30</td>
<td>87 29.2</td>
<td>188.9% 2.8%</td>
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</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>77 28.8</td>
<td>156.8% 4.0%</td>
<td>183.5% 3.0%</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>91 29.4</td>
<td>204.9% 2.2%</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>A</td>
<td>45</td>
<td>141 44.1</td>
<td>212.7% 2.1%</td>
<td>196.6% 2.3%</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>124 43.0</td>
<td>175.8% 4.4%</td>
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</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>136 45.2</td>
<td>201.4% 0.4%</td>
<td></td>
</tr>
<tr>
<td>45B</td>
<td>A</td>
<td>45</td>
<td>191 37.4</td>
<td>324.5% 16.8%</td>
<td>316.0% 18.2%</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>174 37.2</td>
<td>285.7% 17.4%</td>
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</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>197 35.9</td>
<td>337.7% 20.3%</td>
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</tr>
<tr>
<td>31</td>
<td>A</td>
<td>31</td>
<td>88 28.1</td>
<td>183.5% 9.4%</td>
<td>166.2% 6.9%</td>
</tr>
<tr>
<td></td>
<td>B</td>
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<td>75 30.0</td>
<td>143.4% 3.3%</td>
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</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>84 28.5</td>
<td>171.6% 8.0%</td>
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</tr>
<tr>
<td>22</td>
<td>A</td>
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<td>50 24.6</td>
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<td>147.4% 8.6%</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>48 25.0</td>
<td>116.5% 13.8%</td>
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</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>65 22.1</td>
<td>196.6% 0.3%</td>
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</tr>
<tr>
<td>20</td>
<td>A</td>
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<td>42 24.3</td>
<td>109.5% 21.7%</td>
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<tr>
<td></td>
<td>B</td>
<td></td>
<td>40 24.3</td>
<td>102.5% 21.5%</td>
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<tr>
<td></td>
<td>C</td>
<td></td>
<td>50 20.9</td>
<td>148.3% 4.4%</td>
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</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>178.8% 8.3%</td>
</tr>
</tbody>
</table>
Results - 3D Reconstruction
## Error of 3D Reconstruction

<table>
<thead>
<tr>
<th>Vol.</th>
<th>Estimated Vol. (mL)</th>
<th>Estimated No. of NAPL Voxel</th>
<th>Method</th>
<th>Not Recon. NAPL Voxel</th>
<th>Error in NAPL Voxel</th>
<th>Error in Total Voxel</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15.5</td>
<td>37,485</td>
<td>Back Projection</td>
<td>8.6%</td>
<td>11.0%</td>
<td>0.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>7.4%</td>
<td>9.3%</td>
<td>0.3%</td>
</tr>
<tr>
<td>30</td>
<td>29.0</td>
<td>69,869</td>
<td>Back Projection</td>
<td>-2.3%</td>
<td>12.5%</td>
<td>0.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>7.3%</td>
<td>11.6%</td>
<td>0.8%</td>
</tr>
<tr>
<td>45</td>
<td>43.5</td>
<td>104,966</td>
<td>Back Projection</td>
<td>4.7%</td>
<td>11.6%</td>
<td>1.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>8.1%</td>
<td>10.4%</td>
<td>1.1%</td>
</tr>
<tr>
<td>45B</td>
<td>37.3</td>
<td>89,904</td>
<td>Back Projection</td>
<td>5.2%</td>
<td>21.7%</td>
<td>2.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>7.4%</td>
<td>16.1%</td>
<td>1.4%</td>
</tr>
<tr>
<td>31</td>
<td>29.0</td>
<td>71,089</td>
<td>Back Projection</td>
<td>24.8%</td>
<td>32.6%</td>
<td>2.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>6.2%</td>
<td>23.1%</td>
<td>1.6%</td>
</tr>
<tr>
<td>22</td>
<td>24.8</td>
<td>59,789</td>
<td>Back Projection</td>
<td>-8.2%</td>
<td>42.1%</td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>7.1%</td>
<td>26.7%</td>
<td>1.6%</td>
</tr>
<tr>
<td>20</td>
<td>24.3</td>
<td>58,607</td>
<td>Back Projection</td>
<td>10.0%</td>
<td>42.8%</td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3D Carving</td>
<td>13.2%</td>
<td>31.1%</td>
<td>1.8%</td>
</tr>
</tbody>
</table>

Average - Back Projection 21.8% 1.5%
Average - 3D Carving 16.0% 1.1%
## Error of 3D Reconstruction

<table>
<thead>
<tr>
<th>Volume of NAPL</th>
<th>Original Projections</th>
<th>2D Projection of 3D Model</th>
<th>Error Image of projection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3D Carving</td>
<td>Back Projection</td>
</tr>
<tr>
<td>15mL</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>20mL</td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td><img src="image13.png" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Result of 3D Reconstruction
Conclusion - Methodology

✓ Fast, sustainable, non-intrusive, non-interruptive, relatively accurate, and inexpensive alternative to other mapping techniques

✓ 1,000,000 voxels resolution

✓ Estimated the volume of injected dyed NAPL (92% accuracy)

✓ Reconstructed the 3D distribution of NAPL (98% accuracy)
More complex modeling scenario of a contaminant transport due to a point source

Simulate a pump and treat remediation process involving multiple injection and extraction wells under different scenarios

Visualization of settlement due to dewatering using a single and/or multiple wells.

A steady-state condition can also be modeled and different transport mechanisms can be studied using this technology.

Study interaction of liquid oxygen and contamination during bioremediation process using multiple color dye tracers.
Limitations & Suggestions for Improvement

1. The 3D carving reconstruction algorithm assumes that the model consists of hv-convex binary matrices which limit the modeling to a point source contamination. However, by using non-convex optimization techniques which rely on stochastic optimization algorithms more complex inverse problems can be solved.

2. Models are limited to 15x15x15 cm due to impurities within the fused quartz supply, and by reducing this source of error, larger models on the order of 30x30x30 cm could be employed.

3. Moreover, NAPL volume measurements are subject to errors resulting from optical heterogeneity due to spatial variation of refractive index, presence of small air bubbles, as well as the difficulty in obtaining perfect image registration of successive images.

To reduce these errors, one should:
1. clearly mark fiducial point
2. maintain the temperature constant
3. track the volumes and masses during model preparation
4. use a customized calibration model
5. prevent the introduction of bubbles to the model
6. decrease the duration of testing
Publications

Thank You